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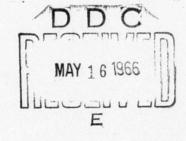
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Numerical Analysis and Related Literature for Scientific Computer Users

Second Edition

James F. Price

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NUMERICAL ANALYSIS AND RELATED LITERATURE FOR SCIENTIFIC COMPUTER USERS

SECOND EDITION

by

James F. Price

Mathematical Note No. 456

Mathematics Research Laboratory

BOEING SCIENTIFIC RESEARCH LABORATORIES

March 1966

PREFACE TO THE SECOND EDITION

Some idea of the book publishing explosion which has been occurring in these subjects in the last five years is obtained by considering that the May 1961 edition of this document listed 69 book titles. This new edition lists 151. In addition, the present list is not quite as complete as the earlier one, and several books reviewed in the earlier edition have now been dropped.

Perhaps the author should have omitted more books from this listing than he did, so that the reader wouldn't have such an imposing volume to look at. However, it is thought that anyone wishing a shorter list can just consider books which are listed with double asterisks (or in the index he can look at underlined page numbers).

The original edition had a section devoted to Numerical Procedures in Recent Periodicals. Such a section has not been included in the Second Edition. The abstracting journal, Computing Reviews, is now operating in a reasonably current manner, so that new literature may now be found quite easily. Periodically (perhaps every three years?), a Permuted (KWIC) Index to Computing Reviews is published which lists journal articles by subject matter (as well as by author and by review number).

NUMERICAL ANALYSIS AND RELATED LITERATURE FOR

SCIENTIFIC COMPUTER USERS

· Second Edition

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ABSTRACT

The Second Edition of this annotated bibliography lists the contents of over 150 books in English on numerical analysis and related literature. It is meant for the general scientific computer user and not for the research numerical analyst; the descriptions and suggestions are given with this in mind.

It is expected that the most useful section will be the 27-page index which tells in which books various topics may be found. There is also a section describing how to look up further information on such topics which may be found in the literature.

INTRODUCTION

When someone plans to put a numerical problem on a digital computer, he may very likely look up the pertinent numerical analysis subject in the reference book on his desk. If this book does not cover the subject, he can go to the library. If the programmer knows the contents of the various reference works, he can run down a particular reference book even if someone else has it checked out at the time. However, most of the huge number of people programming for the digital computers are not specialists in numerical analysis. This document is written to aid such people by (1) listing what is in some of the books on this and related subjects, (2) giving one man's opinion of the relative reading difficulty of various works and of the relative usefulness in computer applications, and (3) explaining how to find out if what they want is available in the literature.

There has been no attempt to make this a complete listing of where information can be obtained on these subjects. Rather it is a listing of reference works which the author and analysts and computer programmers of the author's acquaintance use from time to time. No foreign language references have been given. Although the German, Russian, and French languages have their share of excellent references, and although in some cases the information is not available in English, the fact remains that a large majority of computer users and programmers do not know any other language and/or can never be induced to read foreign language references. This document is directed to such "English-limited" readers; we would not want to discourage the foreign language reader from obtaining his references from the most suitable source independent of language. A great many good Russian books have been translated into English and are listed here.

Part I lists books containing numerical analysis and related subjects. When a computer user has a new problem, he might wish to look up some of his favorite references in this list to see if they discuss this new problem or problems similar to it.

The listing contains a great majority of the English texts on numerical analysis, but only a very limited number on what we think of as "related literature." It was often very hard to decide which books to include and which to omit on such topics as (for example) linear programming, or linear algebra, or FORTRAN programming, which are represented in the literature by a very large number of titles. In many cases the actual choice was quite arbitrary.

Books listed by a double asterisk are recommended as references which the author has found to be of most use to computer people because of their content, their emphasis on computer applications, and/or to their clarity of exposition.

Part II consists of a general discussion of how one goes about looking up given subjects in the literature and how he can find whether or not certain tables are available.

Part III is a subject index. It is thought that this index may be useful either in the obvious way of finding a book which discusses the desired topic, or also in determining whether a certain reference work (which had previously been found especially useful) also is listed as discussing today's topic.

The formation of a really comprehensive index on numerical procedures and related subjects would have been a major undertaking.

Many individual books themselves have only sketchy indes. In general,

this index contains quite specific listings in numerical analysis topics but usually only general listings on related topics. For example, there is no reference made to "Weinstein method," although there is a general reference to "Variational methods"; there is no reference to "Duality," although there are references to other linear programming topics such as "Simplex method."

Often a certain subject may be in a particular book under a different name or under no name at all. Then the question is whether this topic should be shown as being in the book (because it might be so hard to find there). In general, if there is a certain listing in our index, and if although the subject is discussed in a particular source, there is no similar listing in the source's index or paragraph headings, this reference will most often be omitted from our index.

A few general topics are listed with a great many references. The more detailed listings have been chosen quite arbitrarily, but it is hoped that enough such topics are represented in the index to make it of practical use.

Perhaps the most serious complaint about the index may be that while in some cases all of the sources which discuss a certain subject may not be listed, in other cases a source may be listed which only casually refers to the subject. For this reason, when more than five references are given on any subject, a few of them will be underlined as especially recommended.

* * * *

Many pages in this document are partially blank. It is recommended that when new reference works come out which seem useful, they be entered (along with a few comments) in one of the blank spaces; then the new references may be recalled at a later time if the need arises. Finally, for those who feel that the list of references in this document is so long that it is frightening, it is suggested that a shorter list is easily obtained by considering only those titles which are marked with a double asterisk. These are references which are particularly popular with scientific computer users or which the present author feels should be popular. Similarly, the underlined references in Part III make up a smaller group of subject references.

PART I.

NUMERICAL PROCEDURES IN BOOKS

Alphabetical by Author

(A double asterisk ** by an author's name means that the reference is considered especially good and/or popular for the average scientific computer user who is required to learn about the subject.)

BLANK PAGE

ACHIESER, N. I.: Theory of Approximation. Ungar (1956) 307 pp.

This quite advanced treatise was translated from the 1947 Russian edition. The language of functional analysis is used throughout, and the approximation techniques described have been kept as general as possible. This has been considered to be an excellent book, and it is still quoted extensively even though newer treatises are now available.

Contents

- Approximation problems in Linear Normalized Spaces
 This includes the approximation of a function in terms of another function or functions which depend on certain parameters. The classical Weierstrass approximation theorem is an example
- 2. Tchebycheff's domain of ideas
 This includes various approximation problems in which it is desired to minimize the deviation of greatest absolute value
- 3. Elements of Harmonic Analysis
- 4. Certain extremal properties of integral transcendental functions of the exponential type Bernstein's inequality; the theorems of Fejér and Riesz; criterion for representation of continuous functions as Fourier-Stieltjes integrals
- 5. Questions regarding the best harmonic approximation of functions
- 6. Wiener's theorem on approximation

Various addenda and problems:

- A. Elementary extremal problems and certain closure criteria
- B. Szegö's theorem and some of its applications
- C. Further examples of closed sequences of functions
- D. Carathéodory-Fejér problem and similar problems
- E. Solotareff's problems and related problems
- F. The best harmonic approximation of the simplest analytic functions

⁽A double asterisk ** by an author's name means that the reference is considered especially good and/or popular for the average scientific computer user who is required to learn about the subject.)

ADLER, B., FERNBACH, S., and ROTENBERG, M. (Ed.): Methods in Computational

<u>Physics</u>. Academic Press (1963) <u>1</u>, 304 pp.; (1964) <u>2</u>, 271 pp.; (1964)

3, 386 pp; (1965) 4, 385 pp.

The authors state that the major aim of this series of books is to describe various numerical techniques developed recently for the solution of practical physical problems in certain fields by means of digital computers. These books should be very welcome indeed to the computer programmer-analyst who is discouraged by the fact that numerical analysis textbooks usually contain only (and always the same) very simple examples of numerical problems. At least if his field of work is one of those discussed here, he now has some excellent reading material available.

Contents

Volume 1, Statistical Physics

Numerical theory of neutron transport. Monte Carlo methods for solving various practical problems in statistical physics.

Volume 2, Quantum Mechanics

The Gaussian function in calculations of statistical mechanics and quantum mechanics. Atomic self-consistent field calculators by the expansion method. The evaluation of integrals by the Zeta-function expansion. Integrals for diatomic molecular calculations. Nonseparable theory of electron-hydrogen scattering. Estimating convergence rates of variational calculations.

Volume 3, Fundamental Methods of Hydrodynamics

Two-dimensional Lagrangian hydrodynamic difference equations. Mixed Eulerian-Lagrangian method. The strip code and the jetting of gas between plates. CEL: a time-dependent, two-space dimensional, coupled Eulerian-Lagrange code. The tensor code. Calculation of elastic-plastic flow. Solution by characteristics of the equations of one-dimensional unsteady flow. The solution of two-dimensional hydrodynamic equations by the method of characteristics. The particle-incell computing methods for fluid dynamics. The time dependent flow of an incompressible viscous fluid.

Volume 4, Applications in Hydrodynamics

Numerical simulation of the earth's atmosphere. Nonlinear effects of the theory of a wind-driven ocean circulation. Analytic continuation using numerical methods. Numerical solution of complete Krook-Boltzmann equation for strong shock waves. Computer experiments for molecular dynamics problems. Computation of the stability of laminar compressible boundary layer. Some computational aspects of propeller design. Methods of automatic computation of stellar evolution. Computations pertaining to the problem of propagation of a seismic pulse in a layered solid.

AITKEN, A. C.: Determinants and Matrices, 8th ed. Interscience (1956)

Inis is a reasonably elementary text. Numerical procedures are not emphasized, but rather it tells about matrices and determinants in general. Explanations are clear. A ninth edition is supposed to be available now.

- 1. Definitions and fundamental operations of matrices
- 2. Definitions and properties of determinants
- Adjugate and reciprocal matrices; solution of simultaneous equations;
 rank and linear dependence
- 4. Cauchy and Laplace expansions; multiplication theorems
- 5. Compound matrices and determinants; dual theorems
- 6. Special determinants; alternant, persymmetric, bigradient, centrosymmetric, Jacobian, Hessian, Wronskian

ALLEN, D. N. deG.: Relaxation Methods. McGraw-Hill (1954) 257 pp.

This is a reasonably elementary explanation of "how to relax." The author emphasizes that the computer using these methods "should not allow himself to become a human computing machine." He says, "the relaxation method is far from being a method carried out according to any set rules—instead it could be more accurately described as a method of breaking such rules as are necessary formally to explain it." Perhaps this gives an indication as to why these kinds of relaxation methods are not placed on digital computers so very often; certainly this author appears to discourage it.

Contents

- 1. & 2.: Linear algebraic equations
- 3. Framework problems
- 4. Ordinary differential equations
- 5. Laplace's and Poisson's equations
- 6. Normal-gradient boundary conditions
- 7. Quasi-plane-potential equation
- 8. Biharmonic equation
- 9. Simultaneous differential equations
- 10. Triangular nets
- 11. Eigenvalue problems (algebraic equations)
- 12. Eigenvalue problems (differential equations)
- 13. Internal boundaries and interfaces
- 14. Problems involving unknown boundaries
- 15. Non-elliptic differential equations
- 16. Three-dimensional relaxation

Appendixes

- 1. Normalization of algebraic equations
- 2. Standard cases of beam deflection
- 3. Useful finite difference approximations
- 4. Orthogonal properties of normal modes

ALT, F. L.: Electronic Digital Computers. Academic Press (1958) 336 pp.

This is a generally expository book about digital computers and their use in science and engineering. In the part of the book on numerical analysis, various methods are described, often from the point of view of evaluating advantages and disadvantages of particular methods when they are to be carried out on the machine.

General Outline of Contents

- Introduction (generalities concerning computing)
- 2. Automatic digital computers (how the machine works)
- 3. Coding and programming
- 4. Problem analysis:

Errors

Overflow and scaling

Checking

Evaluation of explicit functions

Differentiation and integration

Ordinary differential equations

Partial differential equations

Algebraic equations

Nonlinear equations

Characteristic roots of matrices

5. Matching problems and machines:

Problems from science and engineering

Problems from other areas

Characteristics of machine computation laboratories

ALT, F. L. and RUBINOFF, M. (Ed.): Advances in Computers, Vol. 1. Academic Press (1960)

These volumes (which have been coming out yearly) are "intended to occupy a position intermediate between a technical journal and a collection of handbooks or monographs." Articles are written by experts in various fields, and they survey recent advances in the field. Each article is "intended to be a piece of technical writing intelligible and interesting to specialists in fields other than the writer's own"; in a majority of cases this is probably true.

General Outline of Contents

- Volume 1. General purpose programming for business applications. Numerical weather prediction. Automatic translation of languages. Programming computers to play games. Machine recognition of spoken words. Binary arithmetic.
- Volume 2. Parabolic partial differential equations. Orthonormalizing computation. Microelectronics using electron-beam-activated machining techniques. Linear programming. Theory of automata.
- Volume 3. Computation of satellite orbit trajectors. Multiprogramming. Nonlinear programming. Alternating direction implicit methods. Combined analog-digital techniques in simulation. Information technology and the law.
- Volume 4. Formulation of data processing problems for computers. All-magnetic circuit techniques. Computer education. Digital fluid logic elements. Multiple computer systems.
- Volume 5. Role of computers in election night broadcasting. Automatic programming in eastern Europe. Artificial intelligence and self-organization. Automatic optical design. Computing problems and methods in X-ray chrystallography. Digital computers in nuclear reactor design. Procedure-oriented languages.
- Volume 6. In preparation.

AMES, W. F.: Nonlinear Partial Differential Equations in Engineering.

Academic Press (1965) 511 pp.

This new book will probably be used quite extensively as a reference by engineers and computer programmers who are trying to obtain solutions to such problems. The book discusses the origin of certain equations of this type, some analytic methods, some approximate methods (such as asymptotic processes, perturbation procedures, etc.) and quite a few numerical methods. A book on such a subject is bound to be quite advanced, but theoretical aspects are not stressed, and many examples are given. There is a good list of references at the end of each chapter.

- 1. The origin of nonlinear differential equations
- 2. Transformation and general solutions
- 3. Exact methods of solution
- 4. Further analytic methods
- 5. Approximate methods (perturbation, asymptotic, and weighted residual methods)
- 6. Further approximate methods (integral methods, the maximum operation, series expansions, etc.)
- Numerical methods (for parabolic, elliptic, hyperbolic, and mixed systems)
- 8. Some theoretical considerations (well-posed problems, existence and uniqueness, etc.)

ARDEN, B. W.: An Introduction to Digital Computing. Addison-Wesley (1963) 389 pp.

This is a very elementary textbook introducing the subjects of computing and numerical analysis. The MAD language is used throughout. In the third of the text on numerical methods, "what to do" is emphasized rather than "why."

- 1. Language and notation
- 2. Elements of a practical language
- 3. Statements and flow charts
- 4. Functions and example programs
- 5. The design of a practical machine
- 6. Machine language and components
- 7. Number systems and arithmetic
- 8. Computational error
- 9. Taylor's series and divided differences
- 10. The solution of equations
- 11. Additional programming topics
- 12. Interpolation
- 13. Numerical integration
- 14. Simultaneous linear equations
- 15. Approximation
- 16. The numerical solution of differential equations
- 17. Non-numerical problems
- 18. A simple compiler

BALAKRISHNAN, A.V., and NEUSTADT, L.W. (Ed.): Computing Methods in Optimization Problems. Academic Press (1964) 327 pp.

This book is based on papers given at a conference on the subject in January 1964. The idea was to tell of recent research progress in the area and in particular to indicate computing experience gained on specific large-scale problems. If one were working on a problem similar to one of these, he should certainly be familiar with what is said about it here.

- 1. Variational theory and optimal control theory (Hestenes)
- On the computation of the optimum temperature profile in a tubular reaction vessel (Storey and Rosenbrock)
- 3. Several trajectory optimization techniques (Kopp, McGill, Moyer, Pinkham)
- 4. A steepest ascent trajectory optimization method which reduces memory requirements (Hillsley and Robbins)
- 5. Dynamic programming, invariant imbedding and quasi-linearization: comparisons and interconnections (Bellman and Kalaba)
- 6. A comparison between some methods for computing optimum paths in the problem of Bolza (Faulkner)
- 7. Minimizing functionals on Hilbert space (Goldstein)
- Computational aspects of the time-optimal control problem (Fadden and Gilbert)
- 9. An on-line identification scheme for multivariable nonlinear systems (Hsieh)
- 10. Method of convex ascent (Halkin)
- 11. Study of an algorithm for dynamic optimization (Perret and Rouxel)
- 12. Application of hybrid computers to the iterative solution of optimum control problems (Gilbert)
- 13. Synthesis of optimal controllers using hybrid analog-digital computers (Paiewonsky, Woodrow, Brunner, and Halbert)
- 14. Gradient methods for the optimization of dynamic system parameters by hybrid computation (Bekey and McGee)

BELLMAN, R. and KALABA, R. (Ed.): Modern Analytic and Computational

Methods in Science and Mathematics. Elsevier (1963) 1, 358 pp.;

(1964) 2, 270 pp.; (1965) 3, 220 pp.; 4, in preparation.

These books explain the application of invariant imbedding methods to various practical problems in radiative transfer. The numerical methods used can also be applied to other problems in engineering and physics. A large proportion of the pages in the first two volumes are taken up with giving the FORTRAN programs and the results obtained for particular problems.

- Volume 1, Invariant Imbedding and Radiative Transfer in Slabs of Finite Thickness (Bellman, Kalaba, and Prestrud)
 - 1. The physical model
 - 2. Gaussian quadrature, zeros of Legendre polynomials, approximate system of differential equations, computational details
 - 3. Analytic aspects: existence, uniqueness, convergence
 - 4. Computational results
- Volume 2, Invariant Imbedding and Time Dependent Transport Processes (Bellman, Kalaba, Prestrud, Kagiwada)
 - 1. Numerical inversion of Laplace transforms
 - 2. One dimensional neutron multiplication process
 - 3. Time dependent diffuse reflection from a slab
- Volume 3, Quasilinearization and Nonlinear Boundary Value Problems (Bellman and Kalaba)
 - Application of quasilinearization to the study of the Riccati equation
 - 2. Two-point boundary value problems for second order differential equations
 - 3. Monotone behavior and differential inequalities
 - 4. Systems of differential equations, storage, and differential approximation
 - 5. Partial differential equations
 - 6. Applications in physics, engineering, and biology
 - 7. Dynamic programming and quasilinearization
 - Appendices: Minimum time program; design and control program; radiative transfer program; Van der Pol equation program; orbit determination program; cardiology program.
- Volume 4, Numerical Inversion of the Laplace Transform: with Applications to Biology, Economics, Engineering, and Physics (Bellman, Kalaba, and Lockett
 - Outline: Elementary properties of the Laplace transform; numerical inversion of the Laplace transform; applications; nonlinear equations; dynamic programming and ill-conditioned systems; appendices.

BEREZIN, I. S. and ZHIDKOV, N. P.: <u>Computing Methods</u>, two vols. Addison-Wesley (1963) 464 pp. and 679 pp.

These two volumes are translations from 1959 Russian texts for a two-year (third and fourth year Moscow University) introductory course in numerical methods. In this country it would be senior and first year graduate level. The language of functional analysis is used whenever the authors feel it will help in the explanations, and hence the volumes may be considered as more advanced than they really are. However, what functional analysis they do use is explained from a very practical point of view.

The size of the books and the title lead one to hope that this might be the long awaited single general purpose reference on computing methods, but actually the major portions of these books could have been written before the advent of large scale computers; the title really should have been "Classical Numerical Analysis."

Contents

Volume 1

- 1. Operations on approximate quantities
- 2. The theory of interpolation and certain applications
- 3. Numerical differentiation and integration
- 4. Approximations
- 5. Least squares approximations

Volume 2

- 6. The solution of sets of linear algebraic equations
- 7. Numerical solution of high degree algebraic equations and transcendental equations
- 8. The evaluation of eigenvalues and eigenvectors of matrices
- 9. Approximate methods of solving ordinary differential equations
- 10. Approximate methods of solving partial differential and integral equations

BERGE, C.: The Theory of Graphs and Its Applications. Wiley (1962) 247 pp.

This reasonably elementary book is a translation from a 1958 French edition. The theory of graphs concerns itself with problems which are stated in terms of points and lines or arrows which join them. Examples are communications networks, circuit diagrams, family trees, etc. In addition to explaining underlying theory, this book also gives practical computational algorithms.

- 1. General definitions
- 2. Descendence relations
- 3. The ordinal function and the Grundy function on an infinite graph
- 4. The fundamental numbers of the theory of graphs
- 5. Kernels of a graph
- 6. Games on a graph
- 7. The problem of the shortest route
- 8. Transport networks
- 9. The theorem of the demi-degrees
- 10. Matching a simple graph
- 11. Factors
- 12. Centres of graphs
- 13. The diameter of a strongly connected graph
- 14. The matrix associated with a graph
- 15. Incidence matrices
- 16. Trees and arborescences
- 17. Euler's problem
- 18. Matching in the general case
- 19. Semi-factors
- 20. The connectivity of a graph
- 21. Planar graphs

BICKLEY, W. G. and THOMPSON, R. S. H. G.: <u>Matrices, Their Meaning and</u>
Manipulation. English Univ. Press (1964) 168 pp.

This reasonably elementary text first presents some practical problems which may be formulated in terms of matrices, and then some introductory theory of matrices. The last half of the book is concerned with numerical procedures. There are a wealth of problems, many with answers, so that the text should be suitable for self-study. Most parts of the book are very well written. The index contains only entries which are not mentioned in the very complete table of contents.

- 1. The genesis of matrices
- 2. Matrix notation
- 3. Matrix multiplication
- 4. Some special matrices
- 5. Some applications of matrix products
- 6. Determinants
- 7. Solutions of systems of linear equations, and the reciprocal matrix
- 8. Methods for the numerical solution of systems of linear equations, the inversion of matrices, and the evaluation of determinants
- 9. Eigenvalues and eigenvectors
- 10. Numerical determination of eigenvalues and eigenvectors

BODEWIG, E.: Matrix Calculus, 2nd ed. Interscience (1959) 452 pp.

This is a reasonably elementary book. However, unless one is familiar with his notation, it may often strike one as being "advanced." The subject matter consists mainly of numerical methods in dealing with matrix problems. The author has in mind that digital computers will be used in the work. It is quite a complete exposition of the topics covered.

General Outline of Contents

- Part 1. Matrix calculus (basic definitions and elementary discussion)
- Part 2. Linear equations
 - a. Direct methods of solution
 - 1. Exact solutions
 - Approximate solutions: triangularization, diagonalization, decomposition, Gauss-Doolittle method, etc.
 - b. Iterative methods of solution
- Part 3. Inversion of matrices
 - a. Direct methods
 - b. Iterative methods
- Part 4. Eigenproblems
 - a. Iterative methods (Power method, Jacobi, Magnier's method, Givens' method, gradient method, Wielandt's method, etc.)
 - b. Direct methods (Leverrier's method, Krylov-Duncan method, Hessenberg method, Samuelson method, etc.)

BOOLE, G.: <u>Calculus of Finite Differences</u>, 4th ed. Chelsea (1958) 336 pp.

The first edition of this classical test was published in 1860. A reprint of the second (1872) edition is published by Dover. The third edition (1926) and this so-called fourth edition are practically the same. In the recent literature, references are still often made to this source, although perhaps many times this is done for historical reasons.

- 1. Nature of the calculus of finite differences
- Direct theorems of finite differences (the operators of finite difference calculus)
- 3. Interpolation and mechanical quadrature
- 4. Finite integration and the summation of series
- 5. The approximate summation of series
- 6. Bernoulli's numbers and factorial coefficients
- 7. Convergency and divergency of series
- 8. Exact theorems (remainder terms, etc.)
- 9. Difference equations of the first order
- 10. General theory of the solutions of difference and differential equations of the first order
- 11. Linear difference equations with constant coefficients
- 12. Further miscellaneous propositions and equations; simultaneous equations
- 13. Linear difference equations with variable coefficients
- 14. Mixed (difference-differential) and partial difference equations
- 15. The calculus of functions
- 16. Geometrical applications

BOOTH, A. D.: Numerical Methods, 2nd ed. Academic Press (1957) 195 pp.

This is a reasonably elementary text. A wide range of subjects is covered (usually not in any great detail), but an amazing amount of information is packed into these short chapters.

- 1. Nature and purpose of numerical analysis (6 pages)
- 2. Tabulations and differences (5 pages)
- Interpolation (16 pages)
- 4. Numerical differentiation and integration (23 pages)
- 5. The summation of series (5 pages)
- 6. Solution of ordinary differential equations (16 pages)
- 7. Simultaneous linear equations (38 pages)
- 8. Partial differential equations (32 pages)
- 9. Nonlinear algebraic equations (19 pages)
- 10. Approximating functions (9 pages) (least squares, Tschebycheff polynomials, etc.)
- 11. Fourier synthesis and analysis (10 pages)
- 12. Integral equations (9 pages)

de BRUIJN, N. G.: <u>Asymptotic Methods in Analysis</u>. Interscience (1958) 200 pp.

The application of asymptotic methods to practical problems is quite difficult for the nonspecialist; he will probably find this book to be easier to read than most of the other literature on the subject. However, the methods here are explained by example, and there are no "cook book" formulas to enable one to apply these methods to similar problems without really knowing much about the field.

- 1. Introduction (and definitions)
- 2. Implicit functions
- 3. Summation
- 4. The Laplace method for integrals
- 5. The saddle point method
- 6. Applications of the saddle point method
- 7. Indirect asymptotics
 Tauberian theorems
 Differentiation of an asymptotic formula
- 8. Iterated functions
- 9. Differential equations

BUCKINGHAM, R. A.: Numerical Methods. Pitman (1957) 597 pp.

This is a reasonably elementary textbook. The author says he is concerned mainly with the needs of those using desk computers, but he also has the electronic computers in mind in some cases.

General Outline of Contents

- 1. Interpolation
- 2. Difference methods
- 3. Solution of algebraic equations
- 4. Matrices and determinantal equations
- 5. Solution of ordinary differential equations (initial problems)
- 6. Solution of ordinary differential equations (boundary value problems)
- 7. Integral equations
- 8. Partial differential equations
- Functions of two variables; interpolation, cubature, differentiation, functions of a complex variable
- 10. Fitting data by method of least squares
- 11. Appendices on:
 - A. Relations between powers and factorials
 - B. Summary of difference formulas, with remainders
 - C. Lagrangian formulas for differentiation and integration
 - D. Orthogonal polynomials for curve fitting

BUTLER, R. and KERR, E.: An Introduction to Numerical Methods.

Pitman (1962) 386 pp.

This elementary text could either be used as a self-study book or as a college text in introductory numerical analysis for nonmathematics majors. Although the authors often mention digital computers, their treatment of the subject matter indicates that many times they do not have them in mind.

General Outline of Contents

- 1. Introduction (discussion of errors, etc.)
- 2. The solution of algebraic and transcendental equations (including the solution of simultaneous linear algebraic equations)
- 3. Finite differences
- 4. Interpolation
- 5. Numerical differentiation and integration
- 6. Numerical solution of differential equations (including both initial-value and two-point boundary value problems)

CASHWELL, E. D., and EVERETT, C. J.: A Practical Manual on the Monte

Carlo Method for Random Walk Problems. Pergamon (1959) 153 pp.

This is a reasonably elementary discussion of the Monte Carlo method as applied to problems in atomic physics. Computational details are stressed. In an appendix is a summary of 20 problems which have been coded and run on a digital computer at Los Alamos.

Contents

- 1. Basic principles (pseudo-random numbers)
- 2. The source routine
- 3. The mean free path and transmission
- 4. The collision or escape routine
- '5. The collision routine for neutrons
- 6. Photon collisions
- 7. Direction parameters after collision
- 8. Terminal classification
- 9. Remarks on computation
- 10. Statistical considerations

Appendix: Summary of problems run on Maniac I.

**COLLATZ, L.: <u>The Numerical Treatment of Differential Equations</u>, 3rd (English) ed. Springer (1960) 568 pp.

This is quite an advanced treatise on the subject. However, in most cases the author goes into some detail to explain the processes, and there are many worked examples. The translation of the earlier German edition is generally so good that one does not realize it is a translation.

Contents

- Mathematical preliminaries and some general principles (finite difference and interpolation formulas, Green's functions; some error distribution principles; functional analysis)
- 2. Initial value problems in ordinary differential equations
- 3. Boundary value problems in ordinary differential equations
- 4. Initial and initial/boundary value problems in partial differential equations
- 5. Boundary value problems in partial differential equations
- 6. Integral and functional equations

Appendix: Tables of various types of finite difference formulas used in the solution of differential equations problems

**CONTE, S. D.: <u>Elementary Numerical Analysis</u>: An Algorithmic Approach.

McGraw-Hill (1965) 278 pp.

This elementary (junior level) text differs from a majority of such texts which have been swamping the market in that it is not just a poor carbon copy of the older ones. Although it is basically a numerical analysis text, computer flow charts and FORTRAN IV programs are often given in connection with algorithms for carrying out certain computations.

- 1. Number systems and errors
- 2. The solution of nonlinear equations
- 3. Interpolation and approximation
- 4. Differentiation and integration
- 5. Matrices and systems of linear equations
- 6. The solution of differential equations
- 7. Boundary value problems in ordinary differential equations

CRANDALL, S. H.: Engineering Analysis: A Survey of Numerical Procedures.

McGraw-Hill (1956) 417 pp.

This is a reasonably elementary book, and it is excellent as a cross between an engineering and a numerical analysis book. It is not often that in one book you can find fairly careful mathematical formulations as well as clear engineering explanations.

Contents

- Equilibrium problems in systems with a finite number of degrees of freedom (systems of linear equations)
- 2. Eigenvalue problems for systems with a finite number of degrees of freedom
- Propagation problems in systems with a finite number of degrees of freedom
- 4. Equilibrium problems in continuous systems
- 5. Eigenvalue problems in continuous systems
- 6. Propagation problems in continuous systems

As the author points out, chapters 1-3 discuss the three topics, equilibrium problems, eigenvalue problems, and propagation problems, for <u>lumped parameter systems</u>. Chapters 4-6 discuss the same three topics for <u>continuous systems</u>. This careful classification should be a big aid to a computer programmer who is not very familiar with basic engineering course topics.

CURTISS, J. H. (Ed.): <u>Sixth Symposium in Applied Mathematics</u>. McGraw-Hill (1956) 303 pp.

The subject of this symposium in 1953 was "Numerical Analysis." Nineteen papers were given at the time by various numerical analysists. A majority of the papers are quite advanced. Many are of interest to computer users.

- 1. Computational problems in the theory of dynamic programming (Bellman)
- 2. Some methods for solutions of boundary-value problems in linear partial differential equations (Bergman)
- 3. Computational aspects of certain combinatorial problems (Bruck)
- 4. Numerical results in the shock configuration in mach reflection (Clutterham and Taub)
- 5. Some applications of gradient methods (Fischbach)
- 6. Some qualitative comments on stability in partial differential equations (Frankel)
- 7. Approximations in numerical analysis (Hastings, Hayward, Wong)
- 8. The conjugate-gradient method for solving linear systems (Hestenes)
- 9. Number theory on the SWAC (Lehmer)
- 10. The assignment problem (Motzkin)
- 11. The method of steepest descent (Rosenbloom)
- 12. Function spaces and approximation (Sard)
- 13. Some computational problems in algebraic number theory (Taussky)
- 14. Machine attacks on problems whose variables are permutations (Thompkins)
- 15. Best-approximation polynomials of given degree (Walsh)
- 16. Recent results in numerical methods of conformal mapping (Warschawski)
- 17. Un the asymptotic transformation of certain distributions into the normal distribution (Wasow)
- 18. Error bounds for eigenvalues of symmetric integral equations (Wielandt)
- 19. On the solution of linear systems by iteration (Young)

**DANTZIG, G. B.: <u>Linear Programming and Extensions</u>. Princeton Univ. Press (1963) 621 pp.

This is probably the best known book on linear programming. The subject is taken up from a historial point of view first; then after the main body of the book, two final chapters deal especially with applications. Parts of the book might be considered elementary, but probably most would not. There is a 19-page bibliography.

- 1. The linear programming concept
- 2. Origins and influences
- 3. Formulating a linear programming model
- 4. Linear equation and inequality systems
- 5. The simplex method
- 6. Proof of the simplex algorithm and the duality theorem
- 7. The geometry of linear programs
- 8. Pivoting, vector spaces, matrices, and inverses
- 9. The simplex method using multipliers
- 10. Finiteness of the simplex method under perturbation
- 11. Variants of the simplex algorithm
- 12. The price concept in linear programming
- 13. Games and linear programs
- 14. The classical transportation problem
- 15. Optimal assignment and other distribution problems
- 16. The transshipment problem
- 17. Networks and the transshipment problem
- 18. Variables with upper bounds
- 19. Maximal flows in networks
- 20. The primal-dual method for transportation problems
- 21. The weighted distribution problem
- 22. Programs with variable coefficients
- 23. A decomposition principle for linear programs
- 24. Convex programming
- 25. Uncertainty
- 26. Discrete variable extremum problems
- 27. Stigler's nutrition problem; an example of formulation and solution
- 28. The allocation of aircraft to routes under uncertain demand

DAVIS, P.J.: <u>Interpolation and Approximation</u>. Blaisdell (Ginn) (1963) 393 pp.

This is a fairly advanced text on the subject. After studying such a book, the student should be able to read and understand most of the modern literature on interpolation and approximation.

- 1. Introduction: topics from algebra and analysis
- 2. Interpolation
- 3. Remainder theory (dealing with the question of how good the approximations are which result from interpolation)
- 4. Convergence theorems for interpolatory processes
- 5. Some problems in infinite interpolation: infinite systems of linear equations in infinitely many unknowns
- 6. Uniform approximation: Weierstrass' approximation theorem and generalizations; Berstein polynomials; simultaneous approximation of functions and derivatives; simultaneous interpolation and approximation
- 7. Best approximation: normed linear spaces, convex sets; fundamental problem of linear approximation; uniqueness of best approximation; best uniform (Tschebyscheff) approximation of continuous functions; best approximation by nonlinear families
- 8. Least square approximation
- 9. Hilbert space
- 10. Orthogonal polynomials (both real and complex)
- 11. Theory of closure and completeness
- 12. Expansion theorems for orthogonal functions; Fourier series; Legendre series; complex orthogonal expansions; reproducing kernel functions
- 13. Degree of approximation
- 14. Approximation of linear functionals

DORFMAN, R., SAMUELSON, P.A., and SOLOW, R. M.: <u>Linear Programming and</u>

<u>Economic Analysis</u>. McGraw-Hill (1958) 525 pp.

This book contains a reasonably elementary exposition of some of the basic concepts of linear and nonlinear programming. It is written for the economist who wishes to learn something about linear programming, so all the emphasis is on applications to this field. However, the examples can be understood by all, and it is generally a popular book.

Contents

- 1. Introduction
- 2. Basic concepts of linear programming
- 3. The valuation problem; market solutions
- 4. The algebra of linear programming
- 5. The transportation problem
- 6. Linear programming analysis of the firm
- 7. Application to the firm; valuation and duality
- 8. Nonlinear programming
- 9. & 10.: The statical Leontief system
- 11. Dynamic aspects of linear models
- 12. Efficient programs of capital accumulation
- 13. Linear programming and the theory of general equilibrium
- 14. Linear programming and welfare analysis
- 15. Elements of game theory
- 16. Interrelations between linear programming and game theory

Appendices: Chance, utility and game theory; the algebra of matrices

DUSINBERRE, G. M.: Heat Transfer Calculations by Finite Differences.

International Textbook Co. (1961) 293 pp

This is a reasonably elementary book which has been written primarily for engineers. The author's previous text, "Numerical Analysis of Heat Flow" (which has been out of print for some time) was used for many years as their only numerical analysis reference book by many heat engineers. This newer text is recommended for a computer programmer who wishes to learn a little more of the physical background of the heat flow problems than he has found in standard numerical analysis texts. A text written from a more modern point of view should be consulted when desiring to learn more about stability of the finite difference equations of heat flow.

- 1. Thermal properties
- 2. Fundamental equations
- 3. Alternative formulations
- 4. Analytical and analog methods
- 5 & 6: One-dimensional transient problems
- 7. Multidimensional transient problems
- 8. Transient flow systems
- 9 & 10: Steady state problems
- 11. Variable properties
- 12. Digital computers
- 13. Miscellaneous examples
- 14. Problems

DWYER, P. S.: Linear Computations. Wiley (1951) 344 pp.

This is a quite elementary text on linear problems and on applications to statistics. There is no emphasis on methods for digital computers. Large numbers of worked out examples make it a very easy book to understand.

- 1-3. Introductory material on the use of desk calculators, significant figure theory, and general principles of good computational design
- 4-8. solution of simultaneous linear equations by various methods
- 9-10. Evaluation of determinants
- 11. Evaluation of linear forms
- 12-14. Matrices; the calculation of the adjoint and of the inverse of a matrix
- 15. The characteristic equation, its roots, and the characteristic vectors
- 16. Other methods of solving the problems of Chapters 4-15.
- 17. The errors of linear computations
- 18. Applications to statistics
- 19. Application to nonlinear problems

ENGELI, M., GINSBURG, T., RUTISHAUSER, H., and STIEFEL, E.: Refined

Iterative Methods for Computation of the Solution and the Eigenvalues

of Self-Adjoint Boundary Value Problems. Birkhäuser (1959) 107 pp.

This is a very practical description of methods of solving such problems written by four experts in the field. In addition to describing various methods, they often give comparisons of machine times required. The subject matter is not elementary, but the explanations are reasonably clear.

- 1. The self-adjoint boundary value problem. Problems of Dirichlet and Poisson type; energy on the boundary; eigenvalue problems; biharmonic problems; modes of oscillation of a plate (Stiefel)
- 2. Theory of gradient methods (Rutishauser)
- 3. Experiments on gradient methods (Ginsburg)
 - a. Survey of plate experiments
 - b. Plate problem with coarse grid (Ax + b = 0)
 Steepest descent; Tchebyscheff method; conjugate gradient methods; cgT method; combined method; elimination; computation of residuals
 - c. Determination of eigenvalues of the matrix A; conjugate gradient methods with subsequent QD-algorithm; cgT method with subsequent QD-algorithm
 - d. Same problems with fine grid
 - e. A bar problem
- 4. Overrelaxation (Engeli)
 - a. Theory (general relaxation, overrelaxation, "property A," Young's overrelaxation; different methods)
 - b. Numerical results (plate problem)
 - c. Numerical results (bar problem)
- 5. Conclusions (comparison of results)

**FADDEEV, D. K., and FADDEEVA, V. M.: <u>Computational Methods of Linear</u>
Algebra. Freeman (1963) 621 pp.

This treatise is a translation from the Russian. Many parts of the book are reasonably elementary, but some parts are of intermediate difficulty. It discusses in detail most of the methods which are used by computer people in this field. A 1959 book by the second author on the same subject was much shorter, and this larger book can now be described as the leading reference book on the subject for scientific computer users. The main difficulties are translational (as for example, when the quotient-difference algorithm is described as the division and subtraction algorithm).

- 1. Basic material from linear algebra
- 2. Exact methods for solving systems of linear equations
- 3. Iterative methods for solving systems of linear equations
- 4. The complete eigenvalue problem (finding all eigenvalues, and eigenvectors associated with each eigenvalue)
- 5. The special eigenvalue problem (finding only a few of the eigenvalues and/or corresponding eigenvectors; for example, the power method finds one eigenvalue at a time)
- 6. The method of minimal interations and other methods based on the idea of orthogonalization (for solving systems of equations or for solving the complete eigenvalue problem)
- 7. Gradient iterative methods (for solving systems of equations or for solving the complete eigenvalue problem)
- 8. Iterative methods for solving the complete eigenvalue problem
- 9. Universal algorithms (for solving systems of linear equations)
- 10. Fifty-nine pages of bibliography

FADDEEVA, V. N.: <u>Computational Methods of Linear Algebra</u>. Dover (1959) 252 pp.

This is a reasonably elementary book on matrices from the point of view of computing. It is a translation from the Russian, and it discusses a majority of the methods which are used by computer people in this field. The translator has used the terms "proper number" and "proper vector" rather than the more common eigenvalue, eigenvector; or characteristic number, characteristic value; or latent root, latent vector. The much larger book by Faddeev and Faddeeva on the same subject (published in 1963) is greatly to be preferred.

- 1. Basic material about matrices
- 2. Systems of linear equations: Gauss' method; square-root method; inversion of matrices; partitioning methods; bordering method; escalator method; iteration methods (Seidel method); etc.
- 3. Proper numbers and proper vectors of a matrix (eigenvalues and eigenvectors): methods of Krylov, Samuelson, Danilevsky, Leverrier, Faddeev; the escalator method; interpolation methods; determining the "first" proper number; finding proper numbers next in line

**FLETCHER, A., MILLER, J., and ROSENHEAD, L.: An Index of Mathematical Tables, 2nd ed., two vols. Addison-Wesley (1962) 994 pp.

These volumes comprise an annotated bibliography of mathematical tables which are available in various books and periodicals. From a practical point of view, it can be considered as "complete," although such a stupendous effort could never hope to be complete in any strict sense of the work. In most cases the two volumes will be used together. A particular table may be looked up in Volume 1, which gives the author's name; then the actual reference may be found in the author index of Volume 2.

Contents

- Volume 1. Introduction (telling how the indexes are constructed)
 - Part 1. Index according to functions (and giving the author)

Volume 2.

- Part 2. Bibliography (listed by author and giving the literature reference)
- Part 3. Errors (which have been found in various tables)
- Part 4. Index to Volume 1 (by author)

FORSYTHE, G. E. and ROSENBLOOM, P. C.: <u>Numerical Analysis and Partial</u>

Differential Equations. Wiley (1958) 204 pp.

These are survey articles which attempt in a broad manner to discuss the present state of knowledge (1958) in the two fields of the title. Each survey has an excellent bibliography attached. Since Rosenbloom's article on Linear Partial Differential Equations deals with theoretical aspects of the problems (no numerical methods are discussed), only Forsythe's article will be indexed here. Forsythe's article, entitled "Contemporary State of Numerical Analysis," is written for nonspecialists, and in a reasonably elementary manner. It tells about what is being done and it gives references, but it does not go into many details.

Paragraph Headings

- 1. What is numerical analysis? (4 pp.)
- Areas of numerical analysis (1 p.)
- 3. An older Soviet view of numerical analysis (2 pp.)
- 4. A more recent Soviet view of numerical analysis (1 p.)
- 5. Automatic digital computers (3 pp.)
- 6. Two Soviet computers (1 p.)
- 7. Literature on numerical analysis (1 p.)
- 8. Numerical integration and interpolation (2 p.)
- 9. Approximations of functions (3 pp.)
- 10. Solving linear algebraic equations (6 pp.)
- 11. Solving matrix eigenvalue problems (4 pp.)
- 12. Difference methods for Laplace's equation (7 pp.)

Bibliography (5 pp.)

**FORSYTHE, G. E. and WASOW, W. R.: <u>Finite Difference Methods for</u>
Partial Differential Equations. Wiley (1960) 444 pp.

This book is generally quite advanced, although introductions to various of the topics are given in quite elementary language. The subject is developed from the point of view of the analyst who wishes to "solve" partial differential equations on digital computers. At the end is an excellent bibliography with references up to 1959.

Contents

Introduction to partial differential equations and computers

- Hyperbolic equations in two independent variables (finite difference methods for (a) wave equation, (b) systems of equations, (c) quasilinear equations; integration along characteristics; integration by Adams' method; shock waves)
- Parabolic equations (simplest heat flow problems; linear problems in one and in two variables; convergence and stability; nonlinear problems)
- 3. Elliptic equations (problems from physics and engineering; variational formulations; interface conditions; maximum principle; formulating elliptic difference equation problems; classical theory of solving elliptic difference equations; explicit and implicit overrelaxation methods; discretization and round-off errors; membrane eigenvalue problem; solving elliptic equations on an automatic digital computer)
- 4. Initial-value problems in more than two independent variables (the equation of wave propagation; characteristics in several dimensions; a meteorological forecast problem; Fourier method for difference and differential equations; method of Peaceman and Rachford)

FORT, T.: <u>Finite Difference and Difference Equations in the Real Domain.</u> Oxford (1948) 251 pp.

This book is rather advanced, but probably most will find it easier to read than Milne-Thompson. A little more than half the book is on the topic of difference equations.

- 1. Direct difference operators
- 2. Elementary theory of summation
- 3. The Bernoulli and Euler polynomials and numbers
- 4. Summation formulas
- 5. Stirling's numbers and numerical differentiation
- 6. Interpolation and mechanical quadrature
- 7. The elementary theory of the linear recurrent relation
- 8. Maxima and minima of finite sums
- 9. The general boundary problem
- 10. Sourm-Liouville theory
- 11. The solution of a differential equation as the limit of the solution of a difference equation
- 12. The weighted vibrating string and its limit
- 13. The linear recurrent relation of the first order with periodic coefficients
- 14. The linear recurrent relation of the second order with periodic coefficients
- 15. Orthogonal sets and the development of an arbitrary function
- 16. Oscillatory and nonoscillatory linear difference equations of the second order
- 17. The linear difference equation in a continuous real independent variable

FOX, A.H.: Fundamentals of Numerical Analysis. Ronald Press (1963) 147 pp.

This standard short elementary text is really shorter than one would expect a 147-page book to be, because for some reason the author includes a 37-page table of squares of integers.

Contents

- Introduction (concerning errors)
- 2. Systems of linear algebraic equations
- 3. Nonlinear equations
- 4. Polynomial equations
- 5. Numerical integration
- 6. Ordinary differential equations
- 7. Partial differential equations
- 8. Integral equations

Table of squares of integers 1000-9999

**FOX, L.: An Introduction to Numerical Linear Algebra. Oxford (1965) 327 pp.

This fairly elementary text is a very good place for a computer user to look up various methods of solving systems of equations and of obtaining matrix eigenvalues and eigenvectors (which Fox calls latent roots and latent vectors). In addition to explaining the various methods, the author presents quite a few worked examples.

- 1. Introduction (including a discussion of the use of computing machines)
- Matrix algebra (theory)
- 3. Elimination methods of Gauss, Jordan, and Aitken
- 4. Compact elimination methods of Doolittle, Crout, Banachiewicz, and Cholesky
- 5. Orthogonalization methods
- 6. Condition, accuracy, and precision
- 7. Comparison of methods; measure of work
- 8. Iterative and gradient methods
- 9. Iterative methods for latent roots and vectors
- 10. Transformation methods for latent roots and vectors
- 11. Notes on error analysis for latent roots and vectors

**FOX, L.: <u>Numerical Solution of Two-Point Boundary Problems</u>. Oxford (1957) 371 pp.

This is a reasonably elementary exposition of the subject of ordinary differential equations with two-point boundary conditions. However, some parts are quite advanced. The explanations are particularly clear. Digital computer methods are not stressed.

Partial List of Contents

- 1. The solution of algebraic equations which arise when solving differential equations numerically. Emphasis is on relaxation methods
- 2. Second order equations (various types of boundary conditions)
- 3. First order equations (applying boundary value techniques to problems which are essentially initial value problems)
- 4. Third and fourth order equations
- 5. Eigenvalue problems (including discussion of various differential equation eigenvalue problems as well as methods of solving the corresponding matrix eigenvalue problems)
- 6. Initial value techniques for boundary value problems
- 7. Accuracy and precision of boundary value methods
- 8. Miscellaneous methods: simultaneous differential systems; equations with discontinuous coefficients; deferred approach to a limit; nonlinear eigenvalue problems; semi-analytic methods (collation, least squares, Galerkin)

FOX, L. (Ed.): <u>Numerical Solution of Ordinary and Partial Differential</u> Equations. Pergamon Press (1962) 509 pp.

This book contains a reasonably elementary introduction to the study of the numerical solution of ordinary differential equations, integral equations, and partial differential equations. In addition, there are discussions of many quite complicated practical problems and how to solve them numerically; thus for the computer user who complains that textbooks never consider problems as difficult as those he is faced with, this book should be an answer. Of course, in a book such as this which has many authors, the various sections are not uniformly good.

- 1. Ordinary differential equations (initial value problems; boundary value problems; eigenvalue problems; Chebyshev solutions)
- 2. Integral equations (Fredholm equations of 1st, 2nd, and 3rd types; Volterra type equations; singular equations; integro-differential equations; Hartree-Fock equation)
- 3. Introduction to partial differential equations (hyperbolic, parabolic, and elliptic equations in two dimensions)
- 4. Practical problems in partial differential equations (nuclear reactor problems; one-dimensional unsteady flow; characteristics in 3 dimensions; quasi-linear parabolic equations in more than 2 dimensions; linear transport equation in 1 and 2 dimensions; Monte Carlo methods for neutronics problems; plasma physics problems; numerical weather prediction.

FRAZER, R. A., DUNCAN, W. J., and COLLAR, A. R.: <u>Elementary Matrices</u>.

Cambridge (1938) 416 pp.

This reasonably elementary book has probably been the most popular book on the subject among scientific computer users. More advanced topics are marked with asterisks and these paragraphs may be omitted by the beginner in the field. In the latter part of the book are many applications to problems in dynamics and particularly aerodynamics.

- 1. Funadmental definitions and elementary properties
- 2. Powers of matrices, series, and infinitesimal calculus
- 3. Lambda-matrices and canonical forms
- 4. Miscallaneous numerical methods: solving systems of equations; inverting matrices; finding latent roots and vectors (eigenvalues and eigenvectors)
- 5. & 6.: Linear ordinary differential equations with constant coefficients (systems of equations, boundary value problems, etc.)
- 7. Numerical solutions of linear ordinary differential equations with variable coefficients
- 8. Kinematics and dynamics of systems
- 9. Systems with linear dynamical equations
- 10. Iterative numerical solutions of linear dynamical problems
- 11. Dynamical systems with solid friction
- 12. Illustrative applications of friction theory to flutter problems
- 13. Pitching oscillations of a frictionally constrained aerofoil

FREIBERGER, W. and PRAGER, W. (Ed.): Applications of Digital Computers.

Ginn (1963) 244 pp.

This consists of a set of lectures given in 1961-62 at Brown University on the occasion of establishment of the Brown University Computing Laboratory by 16 well-known authors. Most of these lectures are quite general in nature. The book can be recommended to one who wishes he might have attended such a series of lectures or by one who wishes to get some general ideas of the problems of one of the specific fields discussed.

- 1. Computers and operations research (Morse)
- 2. How computers can learn from experience (Simon)
- 3. Recent developments in the science of diagnosis (Woodbury and Lipkin)
- 4. Recent trends in computer programming and numerical analysis (Carr)
- 5. Using computers to solve problems in physics (Thomas)
- 6. Computers and brains (Rosenblith)
- 7. Sorting on computers (Gotlieb)
- 8. The role of computers in astronomy (Davis)
- 9. Computers in fluid mechanics (Giese)
- 10. The use of digital computers in civil engineering (Massonnet)
- 11. Information theory and numerical analysis (Hamming)
- 12. Educational implications of the computer revolution (Forsythe)
- 13. The analysis and design of experiments with the help of computers (Hartley)
- 14. Automatic data processing in the legal profession (Kehl)
- 15. Automation and pure mathematics (Lehmer)

**FRÖBERG, CARL-ERIK: <u>Introduction to Numerical Analysis</u>. Addison-Wesley (1965) 340 pp.

This is a translation of a Swedish text published in 1962. It is suitable for a first (undergraduate or graduate) course in numerical analysis. This is not just a rehash of methods described in numerical analysis texts of earlier times, but some of the more useful of the newer advances on elementary topics are also considered. Although it may have lost something in its translation, it is still highly recommended. (One can remember, for example, when he is reading about matrices that "regular" means "non-singular," and "quadratic" means "square"). A glance at the contents will suggest, however, that it is not possible to have such a large list of topics covered in as great detail as one might wish if he were to use this only as a reference work.

- 1. Numerical computations (representation of numbers, errors, etc.)
- 2. Equations (methods of finding the roots of equations)
- 3. Matrices
- 4. Linear systems of equations
- 5. Matrix inversion
- 6. Algebraic eigenvalue problems
- 7. Linear operators
- 8. Interpolation
- 9. Numerical differentiation
- 10. Numerical quadrature
- 11. Summation
- 12. Multiple integrals
- 13. Difference equations
- 14. Ordinary differential equations
- 15. Partial differential equations
- 16. Approximation (least square, trigonometric, exponential, Chebyshev, continued fraction, rational)
- 17. The Monte Carlo method
- 13. Linear programming

GARABEDIAN, H. L. (Ed.): Approximation of Functions. Elsevier (1965) 220 pp.

This book contains papers presented at a symposium on the subject in 1964. Almost all are of an advanced nature. The paper by Davis consists mainly of an excellent bibliography compiled in the summer of 1964. Lorentz's paper discusses the Russian literature.

- The convergence of sequences of rational functions of best approximation with some free poles (Walsh)
- 2. Uses of Hilbert space in approximation (Sard)
- 3. Applications of duality in approximation theory (Buck)
- 4. Inclusion theorems for the minimal distance in rational Tschebyscheff approximation with several variables (Collatz)
- 5. Rational approximation on finite point sets (Fox, Goldstein, and Lastman)
- 6. Phase methods for polynomial approximation (Stiefel)
- 7. Optimal and nearly-optimal linear approximation (Golomb)
- 8. Approximation by generalized rational functions (Cheney)
- 9. Nonlinear approximation (Rice)
- 10. Nonlinear sequence transformations (Bauer)
- 11. Approximation theory in the first two decades of electronic computers (Davis)
- 12. Piecewise polynomial interpolation and approximation (Birkhoff and de Boor)
- 13. Russian literature on approximation in 1958-1964 (Lorentz)

GARVIN, W. W.: <u>Introduction to Linear Programming</u>. McGraw-Hill (1960) 281 pp.

This is a reasonably elementary introduction to the subject. There are a great many examples given, and the computational procedures which can be used on a digital computer are emphasized. The author has made a special effort to use only very simple mathematics.

- 1. The general linear programming problem
- 2. The simplex method
- 3. The computational procedure
- 4. Sensitivity analysis
- 5. 10.: Practical problems
- 11. Upper bounds
- 12. Statistical linear programming
- 13. Revised simplex method
- 14. Resolution of degeneracy
- 15. Parametric linear programming
- 16. A simple economic problem
- 17. Duality
- 18. The warehouse problem

GODUNOV, S. K. and RYABENKI, V. S.: <u>Theory of Difference Schemes</u>. North Holland Publ. Co. (1964) 289 pp.

This quite advanced text is a translation from the Russian. Although there is some more general introductory material, the main subject of the book is the theory of difference schemes for partial differential equations. There are many more examples given than in most other books at this level. There is no index.

- 1. Elementary examples of difference schemes of first and second order
- Approximations and stability (difference schemes for ordinary differential equations)
- Concepts of an approximation and stability for difference schemes for partial differential equations
- 4. On the solution of difference equations
- 5. Stability as the boundedness of the powers of a certain operator
- 6. Elements of the spectral theory of difference operators
- Appendix I. On difference schemes for the solution of the equation of thermal conductivity (Gelfond and Lokutsievski)
- Appendix II. The double sweep method for solution of difference equations (Gelfond and Lokutsievski)
- Appendix III. The scope of the energy method (Lax)
- Appendix IV. On the estimate of the amount of computational labor necessary in approximate solutions (Bakhvalov)
- Appendix V. Difference schemes for parabolic equations and continuous integrals (Krylov)

GOULD, S. H.: <u>Variational Methods for Eigenvalue Problems</u>. University of Toronto Press (1957) 179 pp.

This is a fairly advanced book. It employs calculus of variations methods to find bounds for eigenvalues. The theoretical engineering aspects of the subject are stressed, but computational methods are also discussed (as the author states, "variational methods are particularly well adapted to successive approximation").

- 1. Systems vibrating with a finite number of degrees of freedom
- 2. Variational principles for finite-dimensional systems
- 3. Vibration of systems with infinitely many degrees of freedom
- 4. Reproducing kernels and functional completion
- 5. Vibrating rods, membranes, and plates
- 6. The Weinstein method in its original form
- 7. Linear operators in a Hilbert space
- 8. The Weinstein-Aronszajn method for linear operators in Hilbert space
- 9. Application of the preceding method to the differential problem of the vibrating plate
- 10. Application of the approximate methods to general differential problems

GRABBE, E. M., RAMO, S., and WOOLDRIDGE, D. E. (Ed.): Handbook of

Automation, Computers, and Control, Vol. 1. Wiley (1958)

This is one of a three-volume set to be used mainly as a reference work. The various chapters have different authors. Many of the topics at the first of the book are elementary, but often the discussions are so sketchy that the beginner might have trouble learning about the subject here. In the last half of the book the topics are covered in much greater detail. It is written with computer people in mind, and it is certainly a good ready reference for computer users on a variety of related subjects. Adequate references are listed at the end of each chapter.

- 1. General mathematics (212 pp.): sets; algebraic equations; matrix theory; finite difference equations; differential equations; integral equations; complex variables; operational mathematics; Laplace transforms; conformal mapping; Boolean algebra; probability; statistics.
- 2. Numerical analysis (88 pp.): interpolation, curve fitting, differentiation, and integration; matrix inversion and simultaneous equations; eigenvalues and eigenvectors; digital techniques in statistical analysis of experiments; ordinary differential equations; partial differential equations
- 3. Operations research (124 pp.)
- 4. Information theory and transmission (108 pp.)
- 5. Feedback control (420 pp.)

GRAM, C. (Ed.): Selected Numerical Methods for Linear Equations, Polynomial Equations, Partial Differential Equations, and Conformal Mapping. Regnecentralen, Copenhagen (1962) 308 pp.

This collection of papers was prepared by a study group organized at the Danish Institute of Computing Machinery to investigate numerical methods in the areas indicated by the title. The purpose was to evaluate and compare various numerical methods. This has not been done in a complete manner at all (except for the conformal mapping work), but it is interesting to read of the results they did obtain. ALGOL programs are given in some cases. On pages 106-113 there is a nice summary of technical terms and key for classification of partial differential equation problems.

- 1. Linear equations (Anderson and Krarup)
- 2. Partial differential equations (Gram, Naur, Poulsen)
 - a. Linear elliptic equations of second order
 Dirichlet problem and other boundary value problems
 Eigenvalue problems
 - The heat equation
 One-, two-, and three-dimensional problems
 Parabolic nonlinear equations
 - c. Hyperbolic equations
 The wave equation and other second order equations of two independent variables
 Systems of first order equations with two or more independent variables
- 3. Conformal mapping (Anderson, Christiansen, Møller, and Tornehave)

 General theory; integral equation methods; application of the

 Lichtenstein-Gershgorin integral equation; conformal mapping of nearby

 circular regions onto the unit disc using expansions in powers of a

 "small parameter"
- 4. Polynomial equations (Busk and Svejgaard)

GRAVES, R. L. and WOLFE, P. (Ed.): Recent Advances in Mathematical

Programming. McGraw-Hill (1963) 347 pp.

The generally advanced papers which make up this book were presented at a symposium on the subject in June, 1962. The first nine papers deal with general theory; the next four deal with nonlinear programming; the next five deal with stochastic programming. Papers 19 through 22 deal with computational procedures for very large linear programming problems; the next four concern themselves with detailed computational variations of the simplex method for linear programs. Papers 27 through 33 are concerned with applications; the next four papers deal with integer programming; the last six papers are concerned with network flow problems.

Contents

Combinatorial theory underlying linear programs. A mutual primal-dual simplex method. On cone functions. The maximum transform (abstract only). Representations for the generalized inverse of matrices partitioned as A = [U,V] (abstract only). On unimodular sets of vectors. Dual programs. Symmetric dual quadratic programs (abstract only). Orthogonality, duality, and quadratic type problems in mathematical programming (abstract only). Methods of nonlinear programming. On the gradient projection methods of R. Frisch and J. B. Rosen (abstract only). The simplex method for local separable programming. Minimization of indefinite quadratic functions with linear constraints (abstract only). Linear programming under uncertainty. A primal-dual algorithm for convex programming (abstract only). Characterizations by chance-constrained programming. Programming with standard errors in the constraints and the objective (abstract only). Inequalities for stochastic nonlinear programming problems (abstract only). Compact basis triangularization for the simplex method. The simplex method using pseudo-basic variables for structured linear programming problems. Dual and parametric methods in decomposition. Convex partition programming. Experiments in linear programming. Parametric linear programming. putational efficiency in product form LP codes. Some new algorithms for linear programming (abstract only). A formula for ranging the cost of living (abstract only). A stochastic model for programming the supply of a strategic material. Mathematical programming applied to short-term scheduling of interconnected hydro-thermal electric power systems (abstract only). An application of linear programming to the fairing of ships' lines. The simulation of multi-component distillation. Optimal capacity scheduling (abstract only). The personnel assignment problem (abstract only). An algorithm for integer solutions to linear programs. Integer quadratic programming. On diagonalization methods in integer programming (abstract only). An accelerated Euclidean algorithm for integer linear programming. Flows in networks. Multi-commodity network flows (abstract only). Transportation problems with distributed loads (abstract only). Least cost estimating and scheduling with limited resources (abstract only). Mathematical programming solution of traveling salesman examples (abstract only). Network algorithms for combinatorial and discrete variable optimization problems (abstract only).

GREENSPAN, D.: Introductory Numerical Analysis of Elliptic Boundary

Value Problems. Harper and Row (1965) 164 pp.

This book could possibly be used as one of a series of specialized texts for a second year course in numerical analysis. The "lemma-proof, theorem-proof,..." method of exposition of thoretical parts of the subject may annoy the nonmathematician, but there are also many numerical examples which the scientific computer user might be glad to see. There is a 24-page bibliography.

Outline of Contents

- 1. Analytical preliminaries
- 2. The Dirichlet problem for the two-dimensional Laplace equation
- 3. The mixed problem for the two-dimensional Laplace equation
- 4. Dirichlet and mixed problems for more general two-dimensional linear elliptic equations
- 5. Three-dimensional problems with axial symmetry
- 6. Linear elliptic problems in three and more dimensions
- 7. Nonlinear problems

With any Course Manager Carp Carp.

GRINTNER, L. E. (Ed.): <u>Numerical Methods of Analysis in Engineering</u>.

Macmillan (1949) 207 pp.

A PERSONAL PROPERTY.

This book consists of a series of papers by well known practicing engineers on numerical methods which they use on practical (usually quite difficult) problems. The emphasis is on methods which depend somewhat on knowledge of the physical problem. Examples in the fields of heat flow and strength of materials are many.

- 1. Numerical methods based upon physical concepts
 - a. Analysis of continuous frames by distributing fixed-end moments (Hardy Cross)
 - b. Statistical state of stress studied by grid analysis (Grintner)
- 2. Numerical solutions of equations for state of stress
 - c. Numerical solutions of boundary value problems by relaxation methods (Shaw)
 - d. The quest for accuracy in computations using finite differences (Southwell)
 - e. The new approach to the numerical solution of Laplace's equation (Frocht)
- 3. Applications of numerical methods to heat transfer
 - f. Numerical solutions for thermal systems (Boeltner and Tribus)
 - g. Two problems in building heating solved numerically (Dusinberre)
- 4. Surveys and bibliographies of numerical methods
 - h. Successive corrections a pattern of thought (Baron)
 - Numerical methods of analysis of bars, plates, and elastic bodies (Newmark)
 - j. A survey of the approximate solution of two-dimensional physical problems by variational methods and finite difference procedures (Higgins)

GRUENBERGER, F. and JAFFRAY, G.: Problems for Computer Solution.

Wiley (1965) 401 pp.

This paperback textbook contains 92 problems of varying difficulties for possible computer solution. It is assumed that the reader has a little basic knowledge of how to use a computer and is now ready to try to place some simple problems on it. In a scientific computing center, this book might be used in the introductory programming course as a source for student problems.

In most cases, a topic is introduced by a brief statement of a problem, followed by some suggestions as to how to proceed. There are quite a few flow charts, but complete FORTRAN programs are not given; the reader is supposed to furnish these!

GUEST, P.G.: Numerical Methods of Curve Fitting. Cambridge (1961) 422 pp.

This quite elementary text is a sort of cross between a statistics book and a numerical methods book. It is written from the point of view of one who will be using desk calculators or no calculator at all.

General Outline of Contents

Single Variables:

- General theory (probability and frequency; expectation and variance; estimation; weights; moments and cumulants)
- 2. The normal distribution
- 3. Some statistical tests
- 4. Discrete distributions (binomial, Poisson)

Regression Theory and the Straight Line

- 5. Regression curves and functional relationship
- 6. The straight line

Polynomials and Other Curves

- 7. Estimation of the polynomial coefficients
- 8. Standard deviations of the estimates
- 9. The grouping of observations
- 10. Functions which are not polynomials (linear functions; nonlinear functions; harmonic analysis; smoothing)
- 11. General regression and functional relationship problems in several variables
- 12. Further illustrative examples (various practical problems)

HADLEY, G. F.: Linear Programming. Addison-Wesley (1962) 520 pp.

HADLEY, G. F.: Nonlinear and Dynamic Programming. Addison-Wesley (1964)

484 pp.

These quite advanced texts are very popular amongst those who work regularly in these fields. The nonlinear programming book is a sequel to the other and assumes the reader is familiar with the earlier material.

Chapter Subjects in First Book

- 1. Introduction
- 2. Mathematical background
- 3 5: Simplex method
- 6. Resolution of the degeneracy problem
- 7. The revised simplex method
- 8. Duality theory and its ramifications
- 9. Transportation problems
- 10. Network flows
- 11. Special topics
- 12. Applications of linear programming to industrial problems
- 13. Applications of linear programming to economic theory

Chapter Subjects in Second Book

- 1. Introduction
- 2. Mathematical background
- 3. Classical optimization methods and properties of convex functions
- 4. Approximate methods for solving problems involving separable functions
- 5. Stochastic problems
- 6. Kuhn-Tucker theory
- 7. Quadratic programming
- 8. Integer linear programming
- 9. Gradient methods
- 10. & 11.: Dynamic programming

HAMMERSLEY, J. M. and HANDSCOMB, D. C.: Monte Carlo Methods. Wiley (1964) 178 pp.

This is quite an advanced book. Direct simulations are discussed in one chapter; but true Monte Carlo techniques are emphasized (in which, for example, valid results may be obtained with less computation by means of various variance reducing techniques). Both the theory and some applications to specific problems are presented.

- 1. The general nature of Monte Carlo methods
- 2. Short resumé of statistical terms.
- 3. Random, pseudorandom, and quasirandom numbers
- 4. Direct simulation
- 5. General principles of the Monte Carlo method
- 6. Conditioned Monte Carlo
- 7. Solution of linear operator equations
- 8. Radiation shielding and reactor criticality
- 9. Problems in statistical mechanics
- 10. Long polymer molecules
- 11. Percolation processes
- 12. Multivariate problems

**HAMMING, R. W.: <u>Numerical Methods for Scientists and Engineers</u>. McGraw-Hill (1962) 411 pp.

Many senior or first year graduate numerical analysis texts look almost as if they might have been written starting from the same rough notes. It is not true of this book; the author's explanations and subject emphasis are sometimes quite novel. For a person who wishes to review an introductory numerical analysis course he had previously, this book might make the job much more interesting than rereading the original text. The author has written the book with the idea in mind that the reader would be using digital computers to solve his practical numerical problems.

Contents

- 1. The difference calculus
- 2. Roundoff noise
- 3. The summation calculus
- 4. Evaluation of infinite series
- 5. Finite difference equations
- 6. The finite Fourier series
- 7. 9.: Polynomial approximation introduction
- 10. A uniform method for finding formulas
- 11. On finding the error term of a formula
- 12. Formulas for definite integrals
- 13. Indefinite integrals
- 14. Introduction to differential equations
- 15. A general theory of predictor-corrector methods
- 16. Special methods of integrating ordinary differential equations
- 17. & 18.: Least squares theory and practice
- 19. Chebyshev polynomials
- 20. Rational functions
- 21. & 22.: Periodic functions Fourier series
- 23. Nonperiodic functions the Fourier integral
- 24. Linear filters smoothing and differentiating
- 25. Integrals and differential equations
- 26. Exponential approximation
- 27. Singularities
- 28. On finding zeros
- 29. Simultaneous linear algebraic equations
- 30. Inversion of matrices and eigenvalues
- 31. Some examples of the simulation of situations and processes
- 32. Random numbers and Monte Carlo methods

Chapter N+1: The art of computing for scientists and engineers

HARRIS, L. D.: Numerical Methods Using FORTRAN. Merrill (1964) 244 pp.

This very elementary book has been written by an Electrical Engineer. A person with a quite limited mathematical background should have little trouble understanding what is said about the various topics, but in many cases he may easily obtain some wrong ideas because of what has not been said. Only FORTRAN II is described in most parts of the book, although one paragraph in Appendix I tells something of FORTRAN IV.

- 1. Solving numerical problems on the digital computer (very short description of a computer)
- 2. The program (mainly an introductory discussion of the FORTRAN language).
- 3. Iteration in the program subscripted variables and the DO statement.
- 4. Interpolation
- 5. Subdividing the program (subroutines and CALL statements)
- 6. Differentiation and integration
- 7. Numerical solution of ordinary differential equations
- 8. Finding the roots of an equation
- 9. Solutions of simultaneous linear equations
- 10. Appendices give more information about FORTRAN

HARTREE, D. R.: <u>Calculating Instruments and Machines</u>. University of Illinois Press (1949) 138 pp.

This rather elementary book consists essentially of a set of lectures given in 1948 at the University of Illinois. The discussion is mainly about the machines, but scattered throughout are discussions of various types of problems to be placed on the machines. Any user of a digital (or of an analogue computer) should have some idea how problems similar to his might be solved on the other type of computer.

- 1. Introduction
- 2. The differential analyser
- 3. The differential analyser and partial differential equations
- 4. Some other instruments: solution of simultaneous linear algebraic equations, the isograph, Fourier synthesizers, integrating instruments, directors
- 5. Large automatic digital machines
- 6. Charles Babbage and the analytical engine
- 7. Historical development
- 8. Projects and prospects
- 9. High speed automatic digital machines and numerical analysis: iterative methods, simultaneous algebraic equations, ordinary differential equations with one-point boundary conditions, ordinary differential equations

HARTREE, D. R.: Numerical Analysis, 2nd ed. Oxford (1958) 283 pp.

This reasonably elementary text is carefully written. The author points the discussion more toward the desk calculator user than toward the digital computer user, but a final chapter has been included entitled "Organization of Calculations for an Automatic Machine."

- 1. Introduction
- Tools of numerical work and how to use them (desk machines, tables, slide rule, graph paper, etc.)
- 3. Evaluation of formulas (difficulties which may arise and how to avoid them)
- 4. Finite differences
- 5. Interpolation
- 6. Integration (quadrature) and differentiation
- 7. Integration of ordinary differential equations
- 8. Simultaneous linear algebraic equations and matrices
- 9. Nonlinear algebraic equations
- 10. Functions of two or more variables (complex numbers, Laplacian operator; elliptic, parabolic, and hyperbolic partial differential equations)
- 11. Miscellaneous processes (summation of series, harmonic analysis, smoothing, etc.)
- 12. Organization of calculations for an automatic machine

**HASTINGS, C.: Approximations for Digital Computers. Princeton Univ.

Press (1955) 201 pp.

This is a reasonably elementary description of the subject of approximating various functions by functions which are relatively easy to evaluate on digital computers. He deals with "best approximations" in the sense of Chebyshev. Part 1 tells something about "how" the approximations listed in Part 2 were derived. Part 2 contains approximation formulas for 23 functions which one might wish to evaluate in computer problems (for example, arcsin x or

$$\frac{2}{\sqrt{\pi}} \int \frac{x}{0} e^{-t^2} dt, etc.$$

Contents of Part 1

- 1. Concerning best fit
- 2. Linear weights
- 3. An iterative procedure
- 4. Solution of equations
- 5. Chebyshev polynomials
- 6. Concerning weights
- 7. Function with a peak
- 8. Rates of convergence
- 9. Choice of form
- 10. A scoring-camera problem

**HENRICI, P.: Discrete Variable Methods in Ordinary Differential

Equations. Wiley (1962) 407 pp.

This book and its sequel, "Error Propagation for Difference Methods," have become the most popular references on the theory of the numerical solution of ordinary differential equations. The exposition is clear, and the subject matter could probably be considered as of intermediate difficulty.

General Outline of Contents

- Part 1. One-step methods for initial value problems
 - 1. Euler's method for a single equation of first order
 - 2. General methods for a single equation of first order
 - 3. General methods for systems of equations of first order
 - 4. Methods for systems of equations of higher order
- Part 2. Multistep methods for initial value problems
 - 1. Methods for equations of the first order
 - 2. Methods for special equations of second order
- Part 3. Boundary value problems

(Direct methods for a class of nonlinear boundary value problems of second order)

**HENRICI, P.: Elements of Numerical Analysis. Wiley (1964) 328 pp.

This reasonably elementary text is written clearly and from a modern point of view. It should turn out to be a very popular introductory numerical analysis college text.

Contents

Introduction

- 1. Definitions
- 2. Complex numbers and polynomials
- 3. Difference equations

Solution of Equations

- 4. Iteration
- 5. Iteration for systems of equations
- 6. Linear difference equations
- 7. Bernoulli's method
- 8. The quotient-difference algorithm

Interpolation and Approximation

- 9. The interpolating polynomial
- 10. Construction of interpolating polynomial: methods using ordinates
- 11. Construction of interpolating polynomial: methods using differences
- 12. Numerical differentiation
- 13. Numerical integration
- 14. Numerical solution of ordinary differential equations (initial value problems)

Computation

- 15. Number systems
- 16. Propagation of round-off error

HENRICI, P.: Error Propagation for Difference Methods. Wiley (1963) 73 pp.

This very well written book is of intermediate difficulty. It is a sequel and companion volume to Henrici's book, "Discrete Variable Methods in Ordinary Differential Equations." Here he extends the theory for multi-step methods to the case of systems of equations. Specific formulas with particular coefficients are not given; rather he discusses general types of methods and their convergence and stability properties.

- 1. & 2.: Introduction, definitions, and notations; definition of the class of systems of ordinary differential equations which are to be studied
- 3. Stability, consistency, and convergence; necessary and sufficient conditions for convergence of a multi-step method
- 4. Asymptotic behavior of discretization error; formulas for the error
- 5. Asymptotic behavior of the round-off error; primary and secondary error; statistical analysis of errors
- Appendix: Application to a simple linear problem and a simple nonlinear problem (both of these problems of circular motion have the same exact solution)

HERGET, P.: The Computation of Orbits. Published by Author (Cincinnati Observatory) (1948; 2nd printing 1962) 177 pp.

This is a very readable text written by an outstanding authority on the topic. Although only desk calculators are assumed, the author has carried out so many extremely lengthy calculations on desk computers that his methods usually turn out well on digital computers also. Thus the book seems more modern than the 1948 publishing date would indicate.

- The problem and methods of orbit computation; numerical methods and practice
- 2. The calculus of finite differences
- 3. Problems in spherical astronomy. Precession; topocentric corrections and parallax; planetary aberration
- 4. The problem of two bodies. Integration of the equations of motion; vectorial methods; Kepler's laws; elements; position in the orbit, Kepler's equation; nearly parabolic orbits; Lambert's theorem
- 5. The method of Laplace. Solution by series; practical methods; derivation of elements and constants for the equator.
- 6. Methods of Gauss and Olbers. Circular orbits; triangle and sectortriangle ratios; Gibbs' formulas; solution for one or two geocentric distances; ephemeris computation; Lambert's equation; parabolic motion; Olbers' method; conditioned orbit
- 7. Improvement of the orbit. Fundamental equations; correction of position and velocity vectors; closed forms for df and dg; parabolic and conditioned solutions; correction of elements; least squares.
- 8. Special perturbations. Cowell's method; preliminary satellite orbit; approximate perturbations in the elements; ephemerides
- 9. Hansen's method of general perturbations. Series developments for reciprocals of mutual distances; harmonic analysis; multiplication of series; use of stencils; Besselian transformations; integration of T series; secular terms; constants of integration
- 10. Tables

APPLICATION OF THE PROPERTY OF THE

HERRIOT, J. G.: Methods of Mathematical Analysis and Computation. Wiley (1963) 198 pp.

This quite elementary text is supposed to be written especially for structures engineers. However, there are almost no practical applications cited; it is a mathematics and numerical analysis book with a choice of topics which the author thinks should be of importance to engineers. It should be quite useful for self-study.

- 1. Interpolation
- 2. Numerical differentiation and integration
- 3. Roots of equations
- 4. Simultaneous linear equations and matrices (This 60-page chapter starts from scratch in discussing linear equations, vectors, and matrices; numerical methods of solving systems of equations and of obtaining eigenvalues and eigenvectors are emphasized)
- 5. Solution of ordinary differential equations (Theory of ordinary differential equations; Green's functions; numerical solution of ordinary differential equations)
- 6. Fourier series (Theory; applications to heat problems and to a rectangular plate problem)
- 7. Numerical solutions of partial differential equations

HILDEBRAND, F. B.: <u>Introduction to Numerical Analysis</u>. McGraw-Hill (1956) 511 pp.

This reasonably elementary book is a popular text for first courses in numerical analysis. It has a good bibliography as an appendix.

Contents

- 1. Introduction (significant figures, random errors, error bounds, etc.)
- 2. Interpolation with divided differences
- 3. Lagrangian methods
- 4. Finite-difference interpolation
- 5. Operations with finite differences
- 6. Numerical solution of ordinary differential equations
- 7. Least-squares polynomial approximation
- 8. Gaussian quadrature and related topics
- 9. Approximations of various types
- 10. Numerical solution of equations

Appendix: Justification of the Crout reduction

HOHN, F. E.: Applied Boolean Algebra, an Elementary Introduction.

Macmillan (1960) 139 pp.

This is a reasonably elementary book written from the point of view of applications to digital computer design. This (single) book on Boolean algebra is included in the listing of ' is in this document because computer people often ask for a book on applied Boolean algebra and often express disappointment in the somewhat more abstract volumes, of which there are many. The book includes no index, but the table of contents is very complete.

Contents

Introduction (definitions, basic identities, etc.)

- 1. Boolean algebra as a model of combinational relay circuitry
- 2. Boolean algebra as a model of propositional logic
- 3. The Boolean algebra of the subsets of a set
- 4. The minimization problem
- Appendix 1. The binary system of numeration (including binary-to-decimal and decimal-to-binary conversion)
- Appendix 2. Semiconductor logic elements

HOUSEHOLDER, A. S.: <u>Principles of Numerical Analysis</u>. McGraw-Hill (1953) 274 pp.

This book is quite advanced. The author is particularly interested in numerical analysis as applied to problems solved on digital computers. Most people find the book hard reading, but it is considered a good reference book by numerical analysts.

- 1. Discussion of various types of errors and of error analyses
- 2. Matrices and linear equations
- 3. Nonlinear equations and systems of equations
- 4. The proper values and vectors of a matrix (eigenvalues and eigenvectors)
- 5. Polynomial, trigonometric, and exponential interpolation
- 6. More general methods of approximation (finite linear methods, Chebyshev expansions, etc.)
- 7. Numerical integration and differentiation
- 8. The Monte Carlo method

**HOUSEHOLDER, A. S.: The Theory of Matrices in Numerical Analysis.

Blaisdell (1964) 257 pp.

The title of this book gives a perfect description of the contents. It is written from a mathematical point of view, but all the subject matter should be of real importance to the computer user who is trying to solve systems of equations or to find matrix eigenvalues and eigenvectors on a computer (the author uses the terms Proper Values and Vectors). The exposition is at a medium level of difficulty. There is an excellent 44-page bibliography.

- 1. Some basic identities and inequalities
- 2. Norms, bounds, and convergence
- 3. Localization theorems and other inequalities (exclusion theorems; inclusion and separation theorems; minimax theorems; inequalities of Wielandt and Kantorovich)
- 4. The solution of linear systems: methods of successive approximation
- 5. Direct methods of inversion
- 6. Proper values and vectors: normalization and reduction of the matrix (methods of Krylov, Webber-Voetter, Danilevskii, Hessenberg, Lanczos, Samuelson and Bryan, Leverrier, etc.)
- 7. Proper values and vectors: successive approximation (methods of Jacobi, Collar and Jahn; power method; LR and QR transformations)

JACKSON, D.: The Theory of Approximation. Amer. Math. Soc. (1930) 178 pp.

This book is easier to read (less advanced) than most American Mathematical Society Colloquium publications (of which this is Vol. 11). It does not attempt to cover the whole subject; however, it covers theoretical aspects of the topics chosen in a complete and rigorous manner.

- Continuous functions (approximation by trigonometric sums and by polynomials)
- 2. Discontinuous functions; functions of limited variation; arithmetic means (convergence of Fourier series and Legendre series)
- 3. The principle of least squares and its generalizations (trigonometric and polynomial approximation)
- 4. Interpolation (trigonometric and polynomial cases; convergence and degree of convergence und r various hypotheses)
- 5. Introduction to the geometry of function space (a 30-year-old introduction into a field which has rapidly expanded in the meantime)

JENNINGS, W.: First Course in Numerical Methods. Macmillan (1963) 233 pp.

me author feels that "most existing texts are far too voluminous for a short introductory course." This small elementary book thus contains a wide variety of topics with usually only a few short paragraphs about each. Digital computer applications are discussed in the introduction, but the remainder of the book does not appear to be slanted this way.

- 1. & 2.: Evaluation of polynomials and their derivatives
- 3. Linear approximations
- 4. Zeros of functions
- 5. Zeros of polynomials
- 6. Basic sets of polynomials
- 7. Polynomial approximations
- 8. Divided differences
- 9. Ordinary differences
- 10. Polynomial interpolation Method of Lagrange
- 11. The π -factor where

$$\pi(x) = (x-x_0)(x-x_1)...(x-x_n)$$

- 12. Evaluation of functions
- 13. Numerical differentiation Method of Lagrange
- 14. Numerical integration Method of Lagrange
- 15. Remainder in numerical integration
- 16. Gaussian quadrature
- 17. Method of undetermined coefficients
- 18. Initial value problems in ordinary differential equations
- 19. Boundary value problems in ordinary differential equations
- 20. Systems of linear algebraic equations
- 21. Matrix inversion
- 22. Matrix characteristic value problems

JORDÁN, C.: Calculus of Finite Differences, 2nd ed. Chelsea (1950) 652 pp.

This is quite an advanced treatise, and it covers the subject thoroughly. It is excellent as a reference work. Statistical applications are stressed. There is no essential difference beween this edition and the original 1939 edition; thus one expects no emphasis on computer applications.

- On operations (forward, backward, central, and divided differences; generating functions; expansions of functions)
- 2. Functions (factorial, gamma, incomplete gamma, digamma, trigamma, beta, incomplete beta, exponential, tigonometric)
- 3. Inverse operation of differences and means. Sums (indefinite sums, summation by parts, determination of sums)
- 4. Stirling's numbers and applications
- 5. Bernoulli polynomials and numbers
- 6. Euler's and Boole's polynomials. Sums of reciprocal powers
- 7. Expansion of functions; interpolation; construction of tables
- 8. Approximation and graduation (orthogonal polynomials, trigonometric approximation, method of least squares)
- 9. Numerical resolution of equations; numerical integration
- 10. Functions of several independent variables (operations, interpolation)
- 11. Difference equations (various methods of solution)
- 12. Equations of partial differences

KANTOROVICH, L. V., and KRYLOV, V. I.: <u>Approximate Methods of Higher</u>
Analysis. Interscience (1958) 681 pp.

This is quite advanced. It is a translation of the 4th Edition of the highly recommended Russian treatise. No thought is given to computer methods, but rather the authors wish to show mathematically how one can obtain approximate solutions to various types of partial differential equation and integral equation problems.

- 1. Methods based on representation of the solution as an infinite series (Fourier method; infinite systems of equations; use of nonorthogonal series; use of double series; improvement of the convergence of series)
- 2. Approximate solution of the integral equations of Fredholm
- 3. The method of nets (finite difference methods for numerical solution of ordinary and elliptic partial differential equations)
- 4. Variational methods
- 5. The conformal transformation of regions
- 6. Applications of conformal transformation (to Dirichlet problem, Neumann problem, etc.)
- 7. Schwarz's method (Dirichlet problem)

KHABAZA, I M.: Numerical Analysis. Pergamon Press (1965) 242 pp.

This elementary paperback text is one of five in a series entitled, "Higher Mathematics for Engineers and Scientists," which have been written for students with an introductory calculus background. The choice of topics and presentation are standard, with no special applications to problems in engineering.

- 1. Digital computers (general description)
- 2. Desk machines; errors in computations
- 3. Finite difference methods
- 4. Recurrence relations: difference equations; summation of series; solution of algebraic equations
- 5. Numerical solution of ordinary differential equations (initial value problems)
- 6. Matrices; solutions of sets of linear equations; latent roots and vectors
- 7. Relaxation methods
 Solution of linear equations; second order ordinary differential equations with two-point boundary conditions; solution of partial differential equations of Poisson type
- 8. Numerical methods for unequal intervals
 Interpolation; curve fitting and method of least squares; orthogonal polynomials; integration (Gaussian quadrature)

KHOVANSKII, A. N.: The Application of Continued Fractions and Their

Generalizations to Problems in Approximation Theory. Noordhoff

(1963) 212 pp.

In addition to being carefully organized, this book has the added advantage that the translator (Peter Wynn) has himself a very good understanding of the subject; thus there is no roughness in the translation. The original Russian edition came out in 1956. If a computer specialist is interested in learning something of the theory of continued fractions which could be of practical use, this book is probably by far the best source.

- Certain problems in the theory of continued fractions
 Convergents; transformations of continued fractions; transformations
 of series into continued fractions; convergence theory and tests
- 2. Continued fraction expansions of certain functions Binomial functions; $\sqrt[X]{x}$; $\ln x$; c^{X} ; inverse trigometric and hyperbolic functions; $\tan x$ and $\tanh x$; $\int_{0}^{x} \frac{dt}{1+t^{k}}$; hypergeometric functions; Prym's function; incomplete gamma function; etc.
- 3. Further methods for obtaining rational function approximations Derivation of various rational function approximations to certain functions with the help of Obreschkoff's formula; derivation of rational function approximations by means of iteration; rational function approximations for many of the functions mentioned in para. 2
- 4. Generalized continued fractions

 The computation of square roots or the solution of quadratic equations with the help of matrices of second order; the calculation of \mathfrak{n}^{th} roots or the solution of \mathfrak{n}^{th} order polynomial equations with the help of matrices of \mathfrak{n}^{th} order

KOPAL, Z.: Numerical Analysis, 2r.1 ed. Wiley (1961) 594 pp.

This is a reasonably elementary book. However, most of the subjects considered are discussed quite fully, and there are many paragraphs which are noted as being more advanced and which do not have to be read in order to understand later paragraphs. The subject of mechanical quadratures is covered thoroughly, and the chapters on ordinary differential equation boundary value problems and on integral and integro-differential equations give much more information than in most general books on numerical analysis. Appendix 4 gives abscissas and weight coefficients of several formulas for mechanical quadratures.

- 1. Introduction
- 2. Polynomial interpolation
- 3. Numerical differentiation
- 4. Integration of ordinary differential equations
- 5. Boundary value problems: algebraic methods
- 6. Boundary value problems: variational, iterative, and other methods
- 7. Mechanical quadratures
- 8. Numerical solution of integral and integro-differential equations
- 9. Operational methods in numerical analysis

KOROVKIN, P. P.: <u>Linear Operators and Approximation Theory</u>. Hindustan Publishing Co. (1960) 222 pp.

This advanced book is a poor translation from the 1959 Russian edition. The language of functional analysis is used throughout.

- 1. Linear positive functionals and operators
- 2. Order of approximation of functions by means of polynomials
- 3. Characteristic of differential properties of a function with respect to the sequence of its best approximations
- 4. Order of approximation of functions by means of linear positive polynomial operators
- 5. Linear continuous polynomial operators
- 6. Fourier series
- 7. Interpolating polynomials

KRYLOV, V. I.: Approximate Calculation of Integrals. Macmillan (1962)

This fairly advanced book is a translation from a 1959 Russian text. Technical terms are translated correctly (which often is not the case in many other translations). Almost all the material is on the approximate integration of functions of a single variable: both definite and indefinite integrals are considered. Appendices give nodes and coefficients for Gauss, Gauss-Hermite, and Gauss-Laguerre quadrature formulas.

Contents

Preliminary Information

- 1. Bernoulli numbers and Bernoulli operators
- 2. Orthogonal polynomials
- 3. Interpolation of functions
- 4. Linear normed spaces; linear operators

Approximate Calculation of Definite Integrals

- 5. Quadrature sums and problems related to them. The remainder in approximate quadrature
- 6. Interpolatory quadratures
- 7. Quadratures of the highest algebraic degree of precision
- 8. Quadrature formulas with least estimate of the remainder
- 9. Quadrature formulas containing pre-assigned nodes
- 10. Quadrature formulas with equal coefficients
- 11. Increasing the precision of quadrature formulas
- 12. Convergence of the quadrature process

Approximate Calculation of Indefinite Integrals

- 13. Introduction (error; convergence; stability)
- 14. Integration of functions given in tabular form
- 15. Calculation of indefinite integrals using a small number of values of the integrand
- 16. Methods which use several previous values of the integral

Artes 2 halfs 100°

KUHN, H. W., and TUCKER, A. W. (Ed.): <u>Linear Inequalities and Related</u>

Systems. Princeton Univ. Press (1956) 322 pp.

This is a collection of 18 quite advanced papers on the theory of linear inequalities written by recognized authorities in the field. It is No. 38 in the <u>Annals of Mathematics Studies</u>. It includes a good sized bibliography, but no index.

- 1. Dual systems of homogeneous linear relations (Tucker)
- 2. Polyhedral convex cones (Goldman and Tucker)
- 3. Resolution and separation theorems for polyhedral convex sets (Goldman)
- 4. Theory of linear programming (Goldman and Tucker)
- 5. On systems of linear inequalities (Fan)
- 6. Infinite programs (Duffin)
- 7. A prime-dual algorithm for linear programs (Dantzig, Ford, and Fulkerson)
- 8. Marginal values of matrix games and linear programs (Mills)
- 9. Determinateness of polyhedral games (Wolfe)
- 10. On systems of distinct representatives (Hoffman and Kuhn)
- 11. Dilworth's theorem on partially ordered sets (Dantzig and Hoffman)
- 12. On the max-flow min-cut theorem of networks (Dantzig and Fulkerson)
- 13. Integral boundary points of convex polyhedra (Hoffman and Kruskal)
- 14. An extension of a theorem of Dantzig (Heller and Thompkins)
- 15. Neighboring vertices on a convex polyhedron (Gale)
- 16. On a theorem of Wald (Kuhn)
- 17. On the solution of a game-theoretic problem (Thompson)
- 18. The closed linear model of production (Gale)

KUNZ, K. S.: Numerical Analysis. McGraw-Hill (1957) 381 pp.

This is a reasonably elementary textbook. In addition to the standard topics covered in most beginning numerical analysis texts, the author has quite a discussion of numerical solution of partial differential and integral equations.

Contents

- 1. Real roots of an equation
- 2. Roots of polynomial equations
- 3. Finite difference tables and the theory of interpolation
- 4. Central difference interpolation formulas
- 5. Lagrange's interpolation formula and inverse interpolation
- 6. Summation of series
- 7. Numerical differentiation and integration
- 8. 9.: Numerical solution of ordinary differential equations
- 10. Simultaneous equations and determinants
- 11. Interpolation in tables of two or more variables
- 12. 14.: Numerical solution of partial differential equations
- 15. Integral equations

Appendix: Estimation of error in numerical computation

LANCE, G. N.: Numerical Methods for High Speed Computers. Iliffe (1960) 166 pp.

This reasonably elementary book discusses all the most important topics in numerical analysis for the average scientific computer programmer. With a small number of pages allotted to each topic, the discussion is limited in many cases to a few sentences. Also, in several cases, when a good method and another generally inferior method are discussed, there is nothing to indicate that the latter has any disadvantages.

General Outline of Contents

- 1. Introduction (a description of computers and their use)
- 2. Evaluation of functions (square roots; e^X; ln x; trigonometric functions; arctangent; arcsine and arccosine; complete elliptic integrals; Jacobian elliptic functions; Bessel functions)
 Chebyshev polynomials and continued fractions are used in deriving formulas for the above
- 3. Solution of ordinary differential equations
- 4. Matrix methods Calculation of latent roots and vectors (eigenvalues and eigenvectors) Matrix interpretive scheme on a digital computer
- 5. Numerical solution of partial differential equations
- 6. Miscellaneous processes Solutions of polynomial equations; solution of real algebraic or transcendental equations; continued fractions; interpolation; numerical evaluation of definite integrals; inverse Laplace transforms; Aitken's δ^2 process

Appendix: Floating point arithmetic

Inversion of matrices

**LANCZOS, C.: Applied Analysis. Prentice Hall (1957) 539 pp.

This reasonably elementary book emphasizes the use of vectors, matrices, and eigenvalues. Also stressed is the use of Chebyshev polynomials in a wide variety of problems. Explanations are unusually clear, and the author often emphasizes techniques which are particularly useful in digital computer applications.

- 1. Algebraic equations
- 2. Matrices and eigenvalue problems
- 3. Large-scale linear systems
- 4. Harmonic analysis
- 5. Data analysis (least squares, interpolation, smoothing, numerical differentiation, etc.)
- 6. Quadrature methods
- 7. Power expansions (telescoping power series, use of Chebyshev polynomials, use of the τ method, etc.)

LANGER, R. E. (Ed.): On Numerical Approximation. University of Wisconsin Press (1959) 462 pp.

This is an advanced work. It consists of 21 articles presented by invitation at a Numerical Analysis Symposium in 1958 by internationally known numerical analysts. (Six of the 20 authors are from other countries.) Many of the articles include excellent bibliographies on the topics discussed.

Titles of the Articles

- 1. On trinds and problems in numerical approximation (Ostrowski)
- 2. Linear spaces and approximation theory (Buck)
- Operational methods in numerical analysis based on rational approximations (Kopal)
- 4. On the numerical integration of periodic analytic functions (Davis)
- 5. Some new divided difference algorithms in two variables (Salzer)
- 6. Numerical evaluation of multiple integrals (Hammer)
- 7. Optimal approximation and error bounds (Golomb and Weinberger)
- 8. The rationale of approximation (Sard)
- 9. On extremal approximations (Walsh)
- 10. Numerical methods of Tchebycheff approximation (Stiefel)
- 11. Minimax methods in table construction (Fox)
- 12. Existence of essentially nonlinear families suitable for oscillatory approximation (Motzkin)
- 13. On variation diminishing approximation methods (Schoenberg)
- 14. Approximation by functions of fewer variables (Golomb)
- 15. Extremal approximations—a summary (Miller)
- 16. Survey of recent Russian literature on approximation (Buck)
- 17. The quotient-difference and epsilon algorithms (Bauer)
- 18. Some sufficient conditions for the existence of an asymptotic formula or an asymptotic expansion (Rosser)
- 19. The estimation of (power) spectra and related quantities (Tukey)
- 20. Approximation in partial differential equations (Collatz)
- 21. Special polynomials in numerical analysis (Todd)

LAPIDUS, L.: <u>Digital Computation for Chemical Engineers</u>. McGraw-Hill (1962) 407 pp.

This is a reasonably elementary numerical analysis text written by a Chemical Engineer. The examples are supposed to be directed toward problems of direct chemical engineering interest; but usually the problem is originally stated in quite a mathematical form, so the computer programmer will not find much on the background of chemical engineering problems. However, the choice of topics and the degrees of emphasis placed on each indicate their importance in the numerical solution of chemical engineering problems. The number of topics covered is very large for a book of this size.

- 1. Concepts and history of digital computers
- 2. Polynomial approximation
- 3. Ordinary differential equations
- 4. Partial differential equations
- 5. Systems of linear algebraic equations
- 6. Roots of algebraic and transcendental equations
- 7. Further methods of approximation (least squares; orthogonal polynomials; telescoping power series; continued fractions)
- 8. Optimization and control (linear programming, etc.)

LARSSON, R. D.: Equalities and Approximations - with FORTRAN Programming.

Wiley (1963) 158 pp.

This is a very elementary book, emphasizing concepts from "the new mathematics." The fact that numerical approximation and FORTRAN programming are discussed may make the other material more palatable to the parents of the students taking such an "enriched" course in high school or such a freshman liberal arts course in college.

Contents

- 1. Groups
- 2. Matrices
- 3. FORTRAN programming
- 4. Sets with two operations: rings, integral domains, fields
- 5. Inequalities
- 6. Areas
- 7. Numerical and polynomial approximations

In all chapters after the second, sample FORTRAN programs are given

- LEBEDEV, A. V. and FEDOROVA, R. M.: <u>A Guide to Mathematical Tables</u>.

 Pergamon (1960) 586 pp.
- BURUNOVA, N. M.: <u>A Guide to Mathematical Tables</u>, Supplement No. 1.

 Pergamon (1960) 190 pp:

These two volumes (listing various mathematical tables which have been published throughout the world) are translations from the Russian. Only the words have been translated; tabular matter has been reproduced directly from the original. The main Russian edition was published in 1956, while the supplement references some material from sources as late as 1958. If one cannot find reference to a table he desires either in Fletcher, Miller, and Rosenhead, or in Schütte, or here, he might as well assume he is not going to find such a table published before 1960 or 1961.

Contents

- Part 1. Description of the tables (given in chapters set up according to subject matter)
- Part 2. References to subject material discussed in each of the chapters of Part 1.

Author index

(Both the original and the supplement have the above format)

LEVY, H. and BAGGOTT, E. A.: <u>Numerical Solutions of Differential Equations</u>.

Dover (1955) 238 pp.

For many years this book (published in 1934) was alone in its field. It is reasonably elementary, and it discusses both graphical and numerical solutions of initial value problems for ordinary differential equations. At the time it was written, problems of stability of the finite difference formulas had not worried workers in the field as they have more recently.

Contents

Graphical methods

Numerical solution over a limited range

Forward integration of first order equations (7 different methods)

Simultaneous equations and equations of the second and higher orders

(formulas are given for treating several special types of equations)

Special methods applicable to linear differential equations of the second order

LEVY, H. and LESSMAN, F.: <u>Finite Difference Equations</u>. Macmillan (1961) 278 pp.

The first part of this book is reasonably elementary, but some of the later topics are quite advanced. The subject matter has to do with solutions in closed form, the existence of solutions, etc.; the computation of numerical solutions and the stability problems which might arise in these cases are not discussed.

- 1. Elementary difference operations
- 2. Interpolation and extrapolation
- 3. The determination of difference equations (showing various ways in which difference equations arise)
- 4. Linear difference and functional equations with constant coefficients
- 5. The general difference equation of the first order
- 6. Linear difference equations with variable coefficients
- 7. Some applications of difference equations (a problem of continuous beams; problems of moving spheres; a problem of discs on a shaft; voltage drop in a chain of insulators; a problem in optics; a problem of masses on a string; permutation and combination problems; a problem of sequential tests in industrial statistics)
- 8. Difference equations associated with functions of two variables (partial difference equations with constant coefficients; some special cases of more general equations)

LINNIK, Y. V.: Method of Least Squares and Principles of the Theory of Observations. Pergamon (1961) 360 pp.

This is a translation of a 1958 Russian text. It does not discuss much numerical detail, but rather it is written from a statistical point of view. For example, there is quite a discussion of the problem of determining the proper degree polynomial to use in fitting observed data. Most of the book is written in a fairly elementary manner. There is no index.

General Outline of Contents

- Necessary knowledge of algebra (vectors; linear equations; matrices; determinants; quadratic forms)
- Necessary knowledge of the theory of probability (random variables; correlation ellipsoid and ellipsoid of constant variance; distributions of random variables)
- 3. Necessary knowledge of mathematical statistics (estimation of parameters)
- 4. Direct measurements of equal accuracy (point estimates and estimates by confidence intervals; estimation of accuracy)
- 5. Direct observations of unequal accuracy (point estimation of mean and σ^2 ; estimation by confidence intervals)
- 6. Indirect unconditional measurements

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- 7. The estimation of linear forms in fundamental parameters from indirect observations. Theorems of Neyman and David
- 8. Indirect conditional measurements
- 9. Reduction by means of correlates
- 10. Some examples of the analysis of observations in Geodesy
- 11. Estimation of results of direct and reverse bearings
- 12. Parabolic interpolation by the method of least squares
- 13. Some investigations of A. Wald. Line of orthogonal regression and its application
- 14. Miscellaneous additional results (confidence ellipsoids; dependent observations; non-normal error vector; Cauchy's method of reduction of observations)

LOWAN, A. N.: The Operator Approach to Problems of Stability and

Convergence of Solutions of Difference Equations. Scripta Mathematica, New York (1957) 104 pp.

This is quite an advanced study of stability and convergence of the solutions of various difference equations. However, the author uses the same type of analysis on all his problems, so after one understands thoroughly the methods applied to the first simple problem, the remainder of the book is not too difficult. The appendices give alternate proofs or more detailed proofs of the stability and convergence criteria of the main text.

Contents

- 1. & 2.: Stability analysis of various difference schemes for the onedimensional problem in heat conduction
- 3. Convergence of various iteration procedures
- 4. Stability analysis of various difference schemes for two-dimensional problems in heat conduction
- Numerical treatment of the Laplace and Poisson differential equations in two dimensions
- 6. Convergence of solutions of difference equations to the solutions of the corresponding differential equations (for the one-dimensional differential equation of heat conduction)
- 7. Stability and convergence of difference analogs of the differential equation of wave motion
- 8. Stability analysis of difference analogs of the differential equation of heat conduction in case of variable coefficients

Appendices:

- 1. Further applications of the theorems of Gershgorin and Brauer
- 2. The roots of the determinantal equation
- 3. Generalization of the Richardson iteration procedure to twodimensional potential problems for regions bounded by lattice lines
- 4. On high powers of matrices whose moduli of eigenvalues are smaller than unity
- 5. On the stability of the Peaceman-Rachford scheme for arbitrary two-dimensional regions
- 6. On Richardson's second order scheme

LYUSTERNIK, L. A., CHERVONENKIS, O. A., and YANPOL' SKII, A. R.: <u>Handbook</u> for Computing Elementary Functions. Pergamon (1965) 251 pp.

This translation from a 1963 Russian book gives many methods of computing values for various elementary functions. It also gives the actual algorithms used on several particular Soviet computers. For computer programmers it should be a welcome supplement to the well-known book by Hastings, Approximations for Digital Computers, at least until the long-promised book by Hart, $et\ al$, Handbook of Computer Approximation, is actually published.

Contents

- 1. Rational and power functions
- 2. Exponential and logarithmic functions
- Trigonometric, hyperbolic, inverse trigonometric, and inverse hyperbolic functions
- 4. Algorithms used for computing elementary functions on some Soviet computers (Strela, BESM, M-2, M-3, Ural)

Appendices

- 1. Special polynomials and other functions (Gudermannian, harmonic polynomials, hypergeometric function, orthogonal polynomials)
- 2. Numerical tables

MC CORMICK, J. M. and SALVADORI, M. G.: <u>Numerical Methods in FORTRAN</u>.

Prentice-Hall (1964) 324 pp.

This very elementary text is meant for a person who is beginning numerical analysis as well as beginning to use a computer. After a brief introduction to FORTRAN, several chapters of a standard numerical analysis course follow; then the last 55 percent of the book presents actual FORTRAN II programs, flow charts, etc. Their FORTRAN programming does nothing to train the learner to write so as to conserve machine time.

Chapter Headings

- 1. Computers and programming
- 2. Approximate computations
- 3. Differentiation, integration, interpolation, and extrapolation
- 4. Solution of algebraic and transcendental equations
- 5. Simultaneous linear algebraic equations
- 6. Ordinary boundary value problems
- 7. Ordinary initial value problems
- 8. Two-dimensional problems

FORTRAN II Programs for Topics in Above Chapters

- 2. Evaluation of a polynomial; difference tables; Taylor series computation
- 3. First derivative of tabulated function; Simpson's rule; Richardson extrapolations; Lagrangian interpolation; Aitken-Neville interpolation
- 4. Quadratic equation; cubic equation; quartic equation: Newton's method (real or complex coefficients); bisection method.
- 5. Determinant by pivotal condensation; Gaussian elimination; Gauss-Seidel iteration; eigenvalues of 3rd order matrix; largest eigenvalue by iteration.
- 6. Second order boundary value programs; fourth order boundary value problems.
- 7. Euler method; Runge-Kutta method; Milne's method; simultaneous first order equations. Second order equations: Runge-Kutta, Milne, and a special method.
- 8. Laplace's equation; parabolic equation; hyperbolic equation; a two-dimensional boundary value problem
- 9. Further SUBROUTINES: Square root of a complex number; Bessel functions; simultaneous complex equations; matrix inversion; Simpson's rule with interval halving; two-dimensional Simpson's rule

MC CRACKEN, D. D.: A Guide to ALGOL Programming. (1962) 106 pp. A Guide to FORTRAN IV Programming. (1965) 151 pp. Wiley.

These two manuals explain the use of ALGOL and FORTRAN (respectively) in a manner more like a textbook than is the case with the basic reference manuals. There are more sample problems, and there are exercises for the student (with some answers at the back of the book).

The ALGOL is ALGOL-60 as defined in the May, 1960, issue of Communications of the Association for Computing Machinery in "Report on the Algorithmic Language ALGOL 60," edited by Peter Naur. The FORTRAN IV is as specified in IBM manuals.

Each of these books would be preferable for self study over the basic manuals. Particularly in the case of ALGOL, it is extremely useful to have the problems and examples with the answers.

MC CRACKEN, D. D. and DORN, W. S.: Numerical Methods and FORTRAN

Programming. Wiley (1964) 457 pp.

The FORTRAN in this elementary text is FORTRAN II. The topics of numerical analysis and FORTRAN programming are integrated very effectively, and it should be a very good book for a beginner who wishes to study on his own.

Chapter Headings

- 1. Fundamentals of FORTRAN computation
- 2. Errors
- 3. Practical evaluation of functions
- 4. Program development
- 5. Roots of equations
- 6. Numerical evaluation of integrals
- 7. Subscripted variables and the DO statement
- 8. Simultaneous linear algebraic equations
- 9. Functions and specification statements
- 10. Ordinary differential equations
- 11. Partial differential equations
- Appendix 1. Summary of FORTRAN input and output methods
- Appendix 2. Chebyshev polynomials; formulas for telescoping power series; approximations for some elementary functions; Gauss quadrature abscissas and coefficients.

MC MINN, S. J.: <u>Matrices for Structural Analysis</u>. E. & F. N. Spon. Ltd. (1964) 210 pp.

Other textbooks might offer a better first course in matrices. However, if one already possesses some elementary knowledge of the subject, he might wish to read this text to learn something of the problems in civil and mechanical engineering for which matrix methods are useful, and to learn something of the vocabulary used in structures problems by the practicing engineer.

- 1. Determinants algebraic theory
- 2. Matrices algebraic theory
- 3. Determinants arithmetic computations
- 4. Matrices arithmetic computation (matrix multiplication and inversion)
- 5. Influence coefficients and energy theorems
- 6. & 7.: Statically indeterminate structures (flexibility method and stiff-ness method)
- 8. Matrices theory of characteristic equations, latent roots, functions of matrices, etc.
- 9. Matrices computation of latent roots
- 10. Stability of rigid frames
- 11. Further latent root applications

MACON, N.: <u>Numerical Analysis</u>. Wiley (1963) 161 pp.

This quite elementary text is designed for a one-semester Junior introductory course; it could also be used for self study.

- Basic concepts
 Analog and digital computers; flow charts; error analysis
- 2. Approximation of functions by polynomials
- 3. Iterative methods of solving functional equations
- 4. Matrices and systems of linear equations
- 5. Computational methods with matrices
- 6. The characteristic values and characteristic vectors of a matrix
- 7. Interpolation
- 8. Differentiation and integration
- 9. Remainder terms for the integration formulas
- 10. Ordinary differential equations
- 11. Systems of first order (differential) equations
- 12. Difference equations

METROPOLIS, N. C., TAUB, A. H., TODD, J., THOMPKINS, C. B. (Ed.):

Experimental Arithmetic, High Speed Computing and Mathematics,

Proceedings of Symposia in Applied Mathematics, Vol 15. Amer.

Math. Soc. (1963) 396 pp.

This volume contains 28 of the papers given at two April, 1962 symposia. One symposium was entitled, "Experimental Airthmetic," and the other was entitled, "Interactions Between Mathematical Research and High Speed Computing."

- 1. Purposeful and unpurposeful computing (Cohn)
- 2. Eliminating the irrelevant from mechanical proofs (Davis)
- 3. The mechanization of mathematical arguments (Wang)
- 4. Towards more versatile mechanical translators (Irons)
- 5. Information theory and decoding computations (Elias)
- 6. Adaptive neural networks as brain models (Block)
- 7. Computer investigation of orthogonal Latin squares of order ten (Parker)
- 8. Determination of division algebra with 32 elements (Walker)
- 9. How programming difficulties lead to theoretical advances (Dade and Zassenhaus)
- 10. Methods of successive restrictions in computational problems involving discrete variables (Tompkins)
- 11. An experimental study of the simplex method (Kuhn and Quandt)
- 12. Large and nonconvex problems in linear programming (Gomory)
- 13. Some high speed logic (Lehmer)
- 14. Stability questions for some numerical methods for ordinary differential equations (Dahlquist)
- 15. Some applications of the quotient-difference algorithm (Henrici)
- 16. Plane-rotations in floating-point arithmetic (Wilkinson)
- 17. New aspects in numerical quadrature (Bauer, Rutishauser, and Stiefel)
- 18. On Jacobi rotation patterns (Rutishauser)
- 19. Automatic numerical integration of ordinary differential equations (Nordsieck)
- 20. Survey of stability of different schemes for solving initial value problems for hyperbolic equations (Lax)
- 21. Unexpected dividends in the theory of prime numbers (Rosser)
- 22. The particle-in-cell method for numerical solution of problems in fluid dynamics (Harlow)
- 23. Numerical experiments in atmospheric hydrodynamics (Charney)
- 24. The oscillations of the earth and of the atmosphere (MacDonald)
- 25. Few particle experiments in statistical mechanics (Alder)
- 26. An approach to the Ising problem using a large scale fast digital computer (Yang)
- 27. Applied mathematics as used in theoretical chemistry (Hirschfelder)
- 28. The mechanization of science (Hamming)

MEYER, H. A. (Ed.): Symposium on Monte Carlo Methods. Wiley (1956) 382 pp.

This consists of a series of papers given at a Somposium on Monte Carlo methods at the University of Florida in 1954. Many of the papers are quite advanced, but if one wishes to do more than merely simulate a physical problem on the computer (calling this Monte Carlo), he must be prepared to look into the possibilities of finding variance reducing techniques or other short cuts for his problem. The large bibliography contains also many abstracts of the books and articles listed.

Papers

- 1. An introductory note (Marshall)
- 2. Generation of pseudo-random numbers (Taussky and Todd)
- 3. Phase shifts, middle squares, wave equation (Metropolis)
- 4. A general theory of stochastic estimates of the Neumann series for the solution of certain Fredholm integral equations (Albert)
- 5. Neighbor sets for random walks and difference equations (Motzkin)
- 6. Monte Carlo computations (Dismuke)
- 7. Applications of Monte Carlo methods to tactical games (Ulam)
- 8. Conditional Monte Carlo for normal samples (Trotter and Tukey)
- 9. Monte Carlo techniques in a complex problem about normal samples (Arnold, Bucher, Trotter, and Tukey)
- 10. An application of the Monte Carlo metod to a problem in gamma ray diffusion (Berger)
- 12. Application of multiple stage sampling procedures to Monte Carlo problems (Marshall)
- 13. Questionable usefulness of variance for measuring estimate accuracy in Monte Carlo importance sampling problems (Walsh)
- 14. Experimental determination of eigenvalues and dynamic influence coefficients for complex structures such as airplanes (Vickery)
- 15. Use of different Monte Carlo sampling techniques (Kahn)
- 16. A theoretical comparison of the efficiencies of two classical methods and a Monte Carlo method for computing one component of the solution of a set of linear algebraic equations (Curtiss)
- 17. A description of the generation and testing of a set of random normal deviates (Lytle)
- 18. Machine sampling from given probability distributions (Butler)
- 19. A Monte Carlo technique for obtaining tests and confidence intervals for insurance mortality rates (Walsh)
- 20. Experiments and models for the Monte Carlo method (Walther)

MILLER, K. S.: An Introduction to the Calculus of Finite Differences and

Difference Equations. Holt, Rinehart, and Winston (1960) 167 pp.

This is a reasonably elementary text. It does not emphasize computational aspects of the subject.

- The calculus of finite differences (factorial polynomials, Stirling numbers, the sum calculus, etc.)
- 2. Infinite products (gamma and beta functions)
- Bernoulli numbers and polynomials (Euler-Maclaurin summation formula; asymptotic expansions)
- 4. Linear difference equations in the real domain

MILNE, W. L.: Numerical Calculus. Princeton Univ. Press (1954) 393 pp.

This reasonably elementary text is written in a particularly clear manner. It contains many worked examples.

- 1. Simultaneous linear equations
- 2. Solutions of equations by successive approximations (equation in one unknown or systems of equations; complex roots of algebraic equations; solutions of λ -determinants. Methods or successive substitutions)
- 3. Interpolation
- 4. Numerical differentiation and integration
- 5. Numerical solution of differential equations
- 6. 8.: Finite differences (factorial polynomials, binomial coefficients; forward, backward, central, divided, and reciprocal differences; formulas using these differences)
- 9. Polynomial approximations by least squares
- 10. Other approximations by least squares (harmonic analysis, Gram-Charlier approximation, etc.)
- 11. Simple difference equations

MILNE, W. E.: <u>Numerical Solution of Differential Equations</u>. Wiley (1953) 275 pp.

This is a reasonably elementary exposition of the subject. Worked examples make the details of the numerical methods very clear. The author has computer applications in mind. There is an excellent bibliography.

General Outline of Contents

Part 1. Ordinary equations

Predictor-corrector methods (Adams, Milne, etc.)

Runge-Kutta methods

Methods based on higher order derivatives

Analysis of error

Systems of equations

Higher order equations

Two-point boundary problems

Part 2. Partial equations

Explicit methods

Simplest parabolic and hyperbolic equations

Truncation error

Nonlinear equations

General boundary conditions

Linear equations, matrices, eigenvalues (latent roots)

Implicit methods

Elliptic equations

Analysis of errors

Finding eigenvalues (characteristic numbers) of differential systems

MILNE-THOMPSON, L. M.: <u>The Calculus of Finite Differences</u>. Macmillan (1933) 558 pp.

This quite advanced older treatise is often quoted by other authors working in the field.

- 1. Divided differences
- 2. Difference operators
- 3. Interpolation
- 4. Numerical applications of differences
- 5. Reciprocal differences
- 6. The polynomials of Bernoulli and Euler
- 7. Numerical differentiation and integration
- 8. The summation problem
- 9. The psi function and the gamma function
- 10. Factorial series
- 11. The difference equation of the first order
- 12. General properties of the linear difference equation
- 13. Linear difference equation with constant coefficients
- 14. & 15.: Linear difference equation with rational coefficients; operational methods; Laplace's transformation
- 16. Equations whose coefficients are expressible by factorial series
- 17. The theorems of Poincaré and Perron

**NATANSON, I. P.: Constructive Function Theory. Unger (3 vols.)

Vol. 1: Uniform Approximation (1964) 232 pp.

Vol. 2: Approximation in the Mean. (1965) 176 pp.

Vol. 3: Interpolation and Approximation Quadratures. (1965) 176 pp.

These quite advanced volumes are translations of the three parts of a 1949 Russian treatise. The subject deals with the theory behind the approximate representation of functions by simpler functions (as is done extensively on digital computers).

Contents

Volume 1.

- 1. Weierstrass' approximation theorem
- 2. Algebraic polynomials of the best approximation
- 3. Trigonometric polynomials of the best approximation
- 4. Interrelation between structural properties of functions and the degree of their approximation by trigonometric polynomials
- 5. Characterization of the structural properties of a function by the behavior of its best approximation by trigonometric polynomials
- 6. Interrelation between structural properties of functions and their approximation by algebraic polynomials
- 7. Approximation by means of Fourier series
- 8. The sums of Fejér and de la Vallée-Poussin
- 9. The best approximation to analytic functions
- 10. Properties of some analytic expressions as means of approximation

Volume 2.

- 1. The space $L^2_{p(x)}$
- 2. Orthogonal systems
- 3. Linearly independent systems of functions
- 4. General properties of orthogonal polynomials
- 5. The Legendre polynomials
- 6. The Jacobi polynomials
- 7. The moment problem for finite intervals
- 8. Infinite intervals

Volume 3.

- 1. Various methods of interpolation
- 2. Theorems of a negative character
- 3. The convergence of interpolation processes
- 4. Several convergence processes related to interpolation
- 5. & 6.: Approximation quadratures

**NATIONAL PHYSICAL LABORATORY STAFF: Modern Computing Methods, 2nd ed.
Philosopical Library (1961) 170 pp.

This is a reasonably elementary resume of computational methods, both for hand computation and for high speed computer computation. It is considered to be excellent although rather brief, and it is often referred to in the literature. There is a very good bibliography.

Contents

- 1. Linear equations and matrices: direct methods
- 2. Linear equations and matrices: direct methods on automatic computers
- 3. Latent roots and vectors of matrices (eigenvalues and eigenvectors)
- 4. Linear equations and matrices: iterative methods
- 5. Linear equations and matrices: error analysis
- 6. Zeros of polynomials
- 7. Finite difference methods
- 8. Chebyshev series
- 9. Ordinary differential equations: initial-value problems
- 10. Ordinary differential equations: boundary-value problems
- 11. Hyperbolic partial differential equations
- 12. Parabolic and elliptic partial differential equations
- 13. Evaluation of limits; use of recurrence relations
- 14. Evaluation of integrals
- 15. Tabulation of mathematical functions

Bibliography

NIELSEN, K. J.: Methods in Numerical Analysis, 2nd ed. Macmillan (1964) 384 pp.

This is an elementary text. There are many worked examples.

- 1. Fundamentals
- 2. Finite differences
- 3. Interpolation
- 4. Differentiation and integration
- 5. Lagrangian formulas
- 6. Ordinary equations and systems
- 7. Differential and difference equations
- 8. Least squares and their application
- 9. Periodic and exponential functions

NOBLE, B.: Numerical Methods, Vols. 1 and 2. Oliver and Boyd (1964) 372 pp.

These quite elementary volumes emphasize the practical use of numerical analysis without omitting entirely important concepts in an effort to simplify the presentation. If proofs are not given, the explanations are usually mathematically accurate, as far as they go, and the reader should not come away with erroneous impressions.

Contents

Volume 1 (156 pp.)

- 1. Accuracy and error
- 2. Iterative methods, with applications to the solution of equations
- 3. Elementary programming for digital computers
- 4. Simultaneous linear algebraic equations
- 5. Matrix methods
- 6. Eigenvalues and eigenvectors

Voltage 2 (pp. 157-372)

- 7. Finite differences and the approximate representation of functions
- 8. Polynomial interpolation
- 9. Numerical integration and differentiation
- 10. Ordinary differential equations
- 11. Partial differential equations (only 34 pp.)

NORKIN, S. B. (Ed.): The Elements of Computational Mathematics.

This very elementary book is a translation of a Russian text published in 1960. The various chapters were written by six different authors. The text was written for correspondence course students, and it should be suitable for self-study at an elementary level.

- 1. Computation with approximate numbers, and assessment of errors
- 2. Construction of tables of functions
- 3. Approximate solution of equations
- 4. Systems of linear equations
- 5. Interpolation polynomials
- 6. Approximate computation of integrals
- 7. Approximate integration of differential equations

**USTROWSKI, A. M.: Solutions of Equations and Systems of Equations.

Academic Press (1960) 202 pp.

This is a somewhat advanced book, although some parts discuss quite elementary subjects. The author hopes to "bridge over the gap that still exists between 'pure' and 'practical' mathematics." The computer user will find that this book is much more "practical" than it locks at first and probably more practical than many of the more elementary textbooks on numerical analysis (many of the difficulties encountered in solving certain equations on digital computers are explained clearly here).

A second, much enlarged edition is scheduled for publication in March, 1966.

Contents

- 1. Remainder terms of interpolation formulas
- 2. Inverse interpolation; derivatives of the inverse function
- 3-5. Interation
- 6-10. Newton-Raphson method
- 11. Three interpolation points
- 12. Linear difference equations
- 13. n distinct points of interpolation
- 14. n+1 coincident points of interpolation and the Taylor development of the root
- 15. Norms of vectors and matrices
- 16-17. Theorems on the convergence or divergence of products of matrices
- 18. Characterization of points of attraction and repulsion for iterations with several variables

Eleven appendices treat the following subjects: continuity and relative continuity of the roots of algebraic equations; an explicit formula for the n^{th} derivative of the inverse function; analog of the Reguli Falsi for two equations with two unknowns; Steffensen's improved iteration rule; the Newton-Raphson algorithm for quadratic polynomials; some modifications and an improvement of the Newton-Raphson method; rounding off in inverse interpolation; accelerating iterations with superlinear convergence; roots of f(z) = 0 from the coefficients of the development of 1/f(z); continuity of the fundamental roots as functions of the elements of the matrix.

PANOV, D. J.: Formulas for the Numerical Solution of Partial Differential

Equations by the Method of Differences. Ungar (1963) 133 pp.

This reasonably elementary text is a translation from the Russian (probably the 1951 edition). Emphasis is on the numerical solution of Laplace's equation. Although the book is somewhat old for one on the subject, the wealth of worked examples should still be of interest. This volume has the distinction of being the only one reviewed which has neither a table of contents nor an index!

General Outline of Contents

- 1. Introduction
- Interpolation formulas and difference formulas for the approximate calculation of derivatives; accuracy attainable
- 3. Laplace and Poisson equations
 - a. Choice of networks; treatment of boundary; rectangular and triangular networks
 - b. The Dirichlet problem: methods of improving values on the boundary; refinement of convergence; examples
 - c. The Neumann problem and the third boundary value problem; examples
 - d. Solution of Laplace's equation by use of a mapping of the region
 - e. Solution of the Laplace and Poisson equations for an inhomogeneous medium
- 4. Biharmonic equation; reduction to a system of Poisson equations; direct solution; examples
- 5. Thermal conductivity equation (parabolic)
 Formulas for one and two space dimensional problems; mixed type boundary conditions; examples
- 6. Wave and telegraph equations (hyperbolic); finite difference methods
- 7. Quasi-linear hyperbolic systems: the method of characteristics; Cauchy problem; Riemann problem; mixed problems. Approximate solutions by the method of characteristics; examples

PICC Symposium on the Numerical Treatment of Ordinary Differential

Equations, Integral, and Integro-differential Equations. Birkhaüser

(1960 679 pp.

This contains papers presented at an international symposium held in Rome September, 1960. Some papers are in French, German, or Italian, but a large majority of them are in English.

Papers in English Are on the Following Topics

Iterative methods for the numerical solution of ordinary differential equations. Lyapunov-like functions and approximate solutions of ordinary differential equations. Fourier transforms and analytic functions. The solution of integro-differential equations occurring in nuclear collision problems. An iterative method for solving nonlinear integral equations. A numerical approach to some problems of continuum mechanics and related questions of stability. The numerical solution of ordinary differential equations in Chebyshev series. General problems confronting computer centers. On existence, uniqueness, and numerical evaluation of solutions of ordinary and hyperbolic differential equations. A new difference method for solution of certain second order nonlinear differential equations. Mathematical programming and integral equations. Some estimates of spectra of matrices. On the asymptotic behavior of certain one-dimensional flows. Some numerical solutions of two-point eigenvalue equations with one boundary at infinity. On the general solution of some nonlinear differential equations. Numerical integration of ordinary differential equations by difference methods with automatic determination of steplength. The propagation of round-off error in the numerical solution of initial value problems involving ordinary differential equations of the second order. The numerical solution of a singular integral equation. Convergence of the discrete ordinate method for the anisotropic scattering transport equation. On the numerical integration of ordinary differential equations and the determination of error bounds. Numerical solution of the differential equations governing the motion of viscous fluid between two rotating discs. Solution of ordinary differential equations by trigonometric interpolation. Accumulation of errors in approximate calculations. Stability of solutions of differential and integral equations. Solution of integro-differential equations arising in the theory of atomic scattering. Theory of numerical convergence of iterative processes with applications to differential equations. Bistable systems of differential equations. Iterative solution of boundary value problems using remainder terms of Taylor expansion. Numerical solution of the singular integral equation for the charge distribution on a flat rectangular lamina. Solution of the Boltzmann Hilbert integral equation; propagation of sound in a rarefied gas. An integrodifferential equation. Application of computing techniques to nuclear physics. Error analysis for the numerical solution of certain differential equations in gasdynamics. Condition of certain integral equations. A self-consistent one group method for the Boltzmann transport equation in neutronics. On spectrum of a mono-energetic neutron transport operator. Perturbed boundary-value problems and their approximate solution. General report on the numerical treatment of integral and integro-differential equations.

PRAGER, W.: Introduction to Basic FORTRAN Programming and Numerical

Methods. Blaisdell (1965) 214 pp.

This is a very elementary paper-bound text which emphasizes the teaching of FORTRAN programming. The numerical methods discussed are incidental to the basic purpose. However, for a computer programmer who wishes to learn some basic numerical analysis, this text will probably be much preferred over one of the older standard texts in which digital computers are not considered at all.

Contents

In addition to the four chapters scattered through the text explaining the various features of the FORTRAN language, and a chapter on flow charts, there are chapters on each of the following topics in numerical analysis:

Error analysis and control

Computing with polynomials

Interpolation

Quadrature

Solution of systems of linear equations; solution of nonlinear equations or systems of nonlinear equations

Integration of ordinary differential equations (both initial and boundary value problems)

RALL, L. B. (Ed.): Error in Digital Computation, Vols. 1 and 2. Wiley (1965) 336 pp. and 288 pp.

Each of these volumes contains the papers which were presented at symposia on the subject at the Mathematics Research Center, University of Wisconsin, in October 1964 and April 1965, respectively. The first volume also contains an excellent bibliography on the subject of error in digital computation.

A majority of the presentations are from a rather advanced standpoint. However, the actual numerical processes for which error analyses are being given are usually well known; thus it should be possible for the average computer programmer to study through and understand most of these papers quite well.

Contents

Volume 1

- 1. The problem of error in digital computation (Todd)
- 2. Techniques for automatic error monitoring and control (Ashenhurst)
- 3. The automatic analysis and control of error in digital computing based on the use of interval numbers (Moore)
- 4. Error in digital solution of linear problems (Albasiny)
- 5. The propagation of error in the digital integration of ordinary differential equations (Henrici)
- 6. Bibliography on error in digital computation (114 pp.)

Volume 2

- 1. Experimental investigation of unnormalized arithmetic (Ashenhurst)
- 2. Error bounds for computations with continued fractions (Henrici)
- 3. Error bounds for asymptotic expansions of special functions in the complex plane (Olver)
- 4. Error analysis for transformations based on the use of matrices of the form I $-2ww^H$. (Wilkinson)
- 5. Automatic local coordinate transformations to reduce the growth of error bounds in interval computation of solutions of ordinary differential equations (Moore)
- 6. Differential inequalities and error bounds (Schröder)
- 7. Discrete representations of partial differential operators (Young and Dauwalder)
- 8. Upper and lower bounds for solutions of integral equations (Brown)
- 9. Convergence and error bounds for approximate solutions of integral and operator equations (Anselone)
- 10. Applications of functional analysis to error estimation (Collatz)
- 11. Error in the solution of linear programming problems (Wolfe)

**RALSTON, A.: A First Course in Numerical Analysis. McGraw-Hill (1965) 578 pp.

This new senior or first year graduate text will probably turn out to be very popular. The author seems to have made some real effort to leave out a lot of the standard classical numerical analysis that has lost its relative importance due to the advent of digital computers. These topics have been replaced by explanations of quite a few of the newer methods which are now being used extensively to solve practical problems on the computer. Each chapter has an up-to-date bibliography.

- Introduction and preliminaries (concerning error and concerning digital computers
- 2. Polynomial approximation
- 3. Interpolation
- 4. Numerical differentiation, numerical quadrature, and summation
- 5 Numerical solution of ordinary differential equations
- 6. Functional approximation least squares techniques
- 7. Functional approximation minimum-maximum error techniques
- 8. The solution of nonlinear equations
- 9. The solution of simultaneous linear equations
- 10. The calculation of eigenvalues and eigenvectors of matrices

**RALSTON, A. and WILF, H. S. (Ed.): Mathematical Methods for Digital

Computers. Wiley (1960) 293 pp.

This book includes 26 expository articles on various topics of interest to those who wish to place scientific problems on digital computers. Most of the articles are reasonably elementary, although sometimes they cover the subject rather sketchily. Since different authors have written different chapters, there is a great diversity as to the excellence of the presentations. However, there are many truly excellent chapters; since the author even presents flow charts describing the particular process, this book should be of very great use to anyone having anything to do with placing such problems on the computer. it is written for computer programmers and by experts who have actually worked with computers.

- 1. Generation of elementary functions (Kogbetliantz)
- 2. Matrix inversion by direct methods (Orden)
- 3. Solution of linear equations by Gauss-Seidel method (Van Norton)
- 4. Solution of linear equations by conjugate-gradient method (Beckman)
- 5. Matrix inversion by rank annihilation (Wilf)
- 6. Matrix inversion by Monte Carlo methods (Oswald)
- 7. Determination of characteristic roots of a matrix by Jacobi method (Greenstadt)
- 8. Numerical integration of ordinary differential equations (Ralston)
- 9. Runge-Kutta methods for the solution of ordinary differential equations (Romanelli)
- 10. Numerical solution of boundary value problems (Wachspress)
- 11. Solution of ordinary differential equations with large time constants (Certaine)
- 12. Numerical solution of parabolic partial differential equations (Keller)
- 13. Iterative methods for solution of elliptic partial differential equations (Sheldon)
- 14. Monte Carlo method for solution of elliptic partial differential equations (Klahr)
- 15. Numerical solution of hyperbolic partial differential equations by method of characteristics (Lister)
- 16. Numerical solution of hyperbolic partial differential equations by difference methods (Fox)
- 17. Multiple regression analysis (Efroymson)
- 18. Factor analysis (Harman)
- 19. Autocorrelation and spectral analysis (Southworth)
- 20. Analysis of variance (Hartley)
- 21. Numerical solution of polynomial equations (Wilf)
- 22. Methods of numerical quadrature (Ralston)
- 23. Multiple quadrature by Monte Carlo methods (Kahn)
- 24. Fourier analysis (Goertzel)
- 25. Solution of linear programming problems (Arden)
- 26. Network analysis (Bashkow)

REDISH, K. A.: An Introduction to Computational Methods. Wiley (1965)
211 pp.

The author has written this quite elementary book "for the occasional computer user." He thus attempts to present methods which are easy to understand—as opposed to more advanced methods which might be more efficient.

General Outline of Contents

- 1. Introduction (sources of error, etc.)
- 2. Simultaneous linear algebraic equations
- Nonlinear equations (locating roots)
- 4. Finite differences
- 5. Interpolation
- 6. Numerical differentiation and integration
- 7. & 8.: Ordinary differential equations

Isoclinals; graphical methods of solution; use of series

First order equations: predictor-corrector methods; Fox-Goodwin methods, etc.

Second order equations

Initial value methods; boundary value methods

Solution of simultaneous algebraic equations

Boundary conditions at infinity

9. Functions of two variables

Interpolation (both "successive" and "simultaneous"); differentiation
and integration; partial differential equations (with emphasis on
elliptic equations)

10. Miscellanea (a few words concerning each topic)

Approximating functions; difference equations; divided differences; Gauss integration formula; Lagrangian formula; latent roots and vectors; Runge-Kutta methods; singularities; summation of series REMEZ, E. Ya.: General Computational Methods of Chebyshev Approximation.

The Problems with Linear Real Parameters. (2 vols.) U.S. Atomic Energy

Commission Translation Series, AEC-tr-4491 (original Russian ed., 1957),

740 pp.

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This is an advanced treatise on the subject. However, specific numerical algorithms are also described. The translation is not very smooth. The index is located in the second book.

Outline of Contents

- Volume 1, Integral Methods of Effective Chebyshev Approximation and Properties of Solutions of the Generalized Chebyshev Problem
 - 1. One dimensional problem of Chebyshev approximation by means of ordinary polynomials, and its direct generalization
 - Systems of incompatible linear equations (in the usual and generalized sense), and the general problem of the least deviation from zero for a function that depends linearly on a finite number of parameters
 - 3. Certain cases of the Chebyshev problem, solved without successive computational approximation
 - 4. Certain approximation methods and procedures, connected with Chebyshev problems
- Volume 2, Finite Systems of Incompatible Linear Equations and Network Methods of Chebyshev Approximations
 - 5. General introduction; α -algorithm of successive weighted quadratic approximations, and possible different applications of the algorithm
 - 6. Method of successive equalizing descents
 - 7. Method of prevailing deviations

Appendix 1: Convex sets in Euclidean n-space

Appendix 2: Solution of systems of linear inequalities

**RICE, J. R.: <u>The Approximation of Functions</u>. Vol. 1: <u>Linear Theory</u>.

Addison-Wesley (1964) 203 pp.

This fairly advanced text discusses the problem of the approximation of a real continuous function by an approximating function which depends on a fixed finite number of parameters. In most cases discussed, these parameters occur in a linear manner, although "unisolvent" nonlinear approximating functons are also discussed (for example, rational approximations). Theory is emphasized, but computing applications are always in mind, and the reader will often find answers to questions of a practical nature as he goes along.

General Outline of Contents

1. Fundamentals

Approximation problem; L_p norms; Tchebycheff norm and the Polya algorithm; linear spaces; choice of form and norm; existence theorems

- 2. Least squares and orthogonal functions
- 3. Tchebycheff approximation

Characterization of Best approximations; uniqueness, continuous dependence; approximation on finite subsets; the de la Valleé-Pouissin algorithm; unisolvent functions; rational functions; limits of a Tchebycheff type theory

4. Approximation in the L, norm

A convex set K in Euclidean (n+1) space; tangent planes to K; characterization of best L_1 approximations; uniqueness; polynomials and trigonometric sums; finite point sets

- 5. The Weierstrass theorem and degree of convergence (including topics on Fourier series and concerning kernels)
- 6. Computational methods

Expansions from analysis; transformations of known expansions; telescoping procedures; method of descent; descent mapping; method of ascent for Tchebycheff approximation; approximation theory and programming

RICHARDSON, C. H.: An Introduction to the Calculus of Finite Differences.

Van Nostrand (1954) 142 pp.

This is a reasonably elementary textbook. There are many exercises and illustrative examples.

- 1. Introduction (definitions and notation; operators)
- Finite integration and applications (includes summation of series;
 Stirling numbers)
- 3. Bernoulli and Euler polynomials
- 4. Interpolation; approximate integration
- 5. Beta and gamma functions
- 6. Difference equations

RICHTMYER, R. D.: Difference Methods for Initial Value Problems.

Interscience (1957) 238 pp.

This is an advanced book. It is highly recommended, however, to anyone faced with putting such problems on computers.

- 1. Introduction: heat flow problem; finite difference equations; convergence; stability; etc.
- 2. Linear operators; Banach spaces; functional analysis
- 3. Linear difference equations
- 4. Pure initial-value problems with constant coefficients
- 5. Multi-level difference equations
- 6. Diffusion and heat flow (31 pp.): simplest heat equation; variable coefficients; nonlinear problems; problems in several space variables
- 7. Transport equation (44 pp.)
- 8. Sound waves (8 pp.)
- 9. Elastic vibrations (16 pp.)
- 10. Fluid dynamics in one space variable (41 pp.)

SALVADORI, M. G. and BARON, M. L.: <u>Numerical Methods in Engineering</u>,

2nd Ed. Prentice-Hall (1961) 258 pp.

This is a quite elementary text emphasizing the numerical solution of algebraic and transcendental equations and especially differential equations. The authors are engineers and have written the book for engineering students.

Contents of 1st Edition

- The practical solution of algebraic and transcendental equations: real and complex roots of algebraic equations; transcendental equations; linear simultaneous algebraic equations (Gauss, Cholesky, Gauss-Seidel, and relaxation methods)
- 2. Finite differences and their applications
- 3. Numerical integration of initial value problems (Adams, Runge-Fox, Adams-Stormer, Fox methods)
- 4. Numerical integration of ordinary boundary value problems
- 5. Numerical solution of partial differential equations

Partial differential operators in Cartesian coordinates

Numerical double integration

Solution of Laplace's and Poisson's equations

Elastic and plastic torsion

Problem involving $\nabla^4 z$ Two-dimensional characteristic value problems

Solution by separation of variables and finite differences

Membrane vibrations

Pivotal points near curved boundaries

A transient problem in two-dimensional heat flow

Laplacian operator in skew, polar, and triangular coordinates

SANGREN, W. C.: Digital Computers and Nuclear Reactor Calculations.

Wiley (1960) 208 pp.

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This book is written to present the nuclear engineer with an introduction to high-speed nuclear-reactor computations. Anyone with some computer experience will probably find the parts of the book describing machines and programming to be very elementary, while the parts discussing the specialized problems will seem much more difficult.

General Outline of Contents

- 1. Introduction (very general remarks concerning computers and reactors)
- 2. Digital computers (description of such machines)
- 3. Programming
- 4. Numerical analysis: approximations; interpolation; numerical differentiation and integration; algebraic and transcendental equations; ordinary differential equations; matrices and linear equations; partial differential equations; the treatment of interfaces
- 5. A code for fission-product poisoning
- 6. Diffusion and age-diffusion calculations: the criticality problem; the one-group problem and the wave equation; the two-group, two-region diffusion problem; one-, two-, and three-dimensional multigroup difference equations; neutron balance; age-diffusion equation
- 7. Transport equation Monte Carlo
- 8. Additional reactor calculations: kinetics; burnout; reactor physics and shielding; engineering

SARD, A.: <u>Linear Approximation</u>. Mathematical Surveys No. 9. Amer. Math.

This is an advanced treatise on topics which are much more general than the word "linear" in the title might imply to some readers; for example, an 11th degree Gaussian quadrature formula might not be thought of as a linear approximation in one sense, but such formulas can be discussed readily in the framework of approximating a linear (integral) operator by another linear operator.

Contents

- 1. Functionals in terms of derivatives
- Applications: Numerical integration (quadrature), interpolation, numerical differentiation, summation
- 3. Linear continuous functionals on C_n
- 4. Functionals in terms of partial derivatives
- 5. Further applications (to problems similar to those of chapter 2)
- 6. Linear continuous functionals on other spaces
- 7. Functions of m variables

Soc. (1963) 544 pp.

- 8. Factors of operators
- 9. Efficient and strongly efficient approximations
- 10. Minimal response to error
- 11. Step functions
- 12. Stieltjes integrals, integration by parts, functions of bounded variation

SAUL'YEV, V. K.: <u>Integration of Equations of Parabolic Type by the</u> Method of Nets. Macmillan (1964) 346 pp.

The title indicates that the whole 346 pages is concerned with the solution of parabolic partial differential equations by finite difference methods. However, implicit finite difference schemes for equations with two space variables become (at a given time t) the same as would occur in a two-dimensional elliptic problem; more than a third of the book discusses the numerical solution of these elliptic type problems.

The book is a translation from a Russian text published in 1960. An elementary notation is used throughout, although of course much of the subject matter is fairly advanced.

General Outline of Contents

- 1. Construction of the net equations (for parabolic problems)

 Stability; six-point equation; asymmetric net equations; alternating method; method of mean arithmetic; multi-nodal symmetric method; comparisons between explicit and implicit methods; spherical and cylindrical regions; fictitious nodes; bilateral approximations; two-dimensional and multi-dimensional equations; non-uniform nets; multi-step equations; variable and discontinuous coefficients; parabolic equations of higher than second order; nonlinear equations
- 2. Solution of the net equations "One-dimensional" elliptic net equations; direct methods; ill-conditioned net matrices; iterative methods of 1st, 2nd, and nth degrees; variational methods; methods using Chebyshev polynomials; methods of successive displacements; block iteration

**SCARBOROUGH, J. B.: <u>Numerical Mathematical Analysis</u>, 5th ed. Johns Hopkins Press (1962) 594 pp.

This is a very elementary text, and the later editions are excellent. Large numbers of worked examples help to explain the processes. Answers to suggested exercises are given at the end of the book. This book is highly recommended for anyone with a very limited mathematical background. The 5th edition has a new chapter on Newton's interpolation formula for unequal intervals.

Contents of 4th Edition

- 1. The accuracy of approximate calculations
- 2. 4.: Interpolation
- 5. The accuracy of interpolation formulas
- 6. Interpolation with two independent variables and trigonometric interpolation
- 7. Numerical differentiation and integration
- 8. The accuracy of quadrature formulas
- 9. The solution of numerical algebraic and transcendental equations
- 10. Graeffe's root-squaring method for solving algebraic equations
- 11. The numerical solution of ordinary differential equations
- 12. The numerical solution of partial differential equations
- 13. The numerical solution of integral equations
- 14. The normal law of error and the principle of least squares
- 15. The precision of measurements
- 16. Empirical formulas (i.e., formulas whose form is inferred from the results of experiment or observation and in which the parameters are determined from experimental or observational data)
- 17. Harmonic analysis of empirical functions
- 18. Numerical solution of simultaneous linear equations

SCHÜTTE, K.: Index of Mathematics Tables. München, Oldenbourg (1955) 143 pp.

This bilingual index lists about 1200 tables; it is smaller than that of Fletcher, Miller, and Rosenhead, but Schütte says that nearly four-fifths of his tables are not shown in the 1946 edition of Fletcher, Miller, and Rosenhead.

- 1. Numerical and practical calculating
- 2. & 3.: Logarithms of natural and circular functions
- 4. Natural values of circular functions
- 5. Simple functions derived from ordinary functions
- 6. Primes; continued fractions; number theory
- 7. Factorials, gamma functions, exponential and hyperbolic functions, etc.
- 8. Elliptic functions and integrals; spherical, Bessel functions, etc.
- 9. Integral tables and other higher functions
- 10. Tables applicable to physics, chemistry, and other sciences
- 11. Astronomy and astrophysics
- 12. Geodesy and geophysics
- 13. Nautical and aeronautical determination of position
- 14. Meteorology
- 15. Astronautics
- 16. Miscellaneous; formulas and tables of measures, weights, etc.

SHAW, F. S.: Relaxation Methods. Dover (1953) 396 pp.

This is a reasonably elementary exposition of relaxation methods. These methods can be programmed for the digital computer, but the program may become quite complicated if many of the very effective methods of speeding convergence are introduced into the program. In some parts of the book, engineering terminology is used, but in general the treatment is mathematical. A bibliography of papers published from 1935-1949 is included.

- 1. Linear algebraic simultaneous equations
- 2. Finite-difference approximations
- 3. Linear ordinary differential equations
- 4. 6.: Second order linear partial differential equations
- 7. Fourth order linear partial differential equations
- 8. Eigenvalue problems
- 9. Free surface problems, integral equations, accuracy, nondimensional treatment

SINGER, J.: <u>Elements of Numerical Analysis</u>. Academic Press (1964) 395 pp.

This is a quite elementary text in classical numerical analysis. It can be recommended for someone who wishes to solve numerical analysis problems without the use of computers. In particular, no other recent numerical analysis text has so much space devoted to graphical methods of solving equations and systems of equations.

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- 1. Significant figures and errors
- 2. The approximating polynomial; approximation at a point
- 3. The approximating polynomial; approximation in an interval
- 4. The numerical solution of algebraic and transcendental equations in one unknown; geometric methods (construction of scales and rules, nomography, etc.)
- 5. The numerical solution of algebraic and transcendental equations in one unknown; arithmetic methods
- 6. The numerical solution of simultaneous algebraic and transcendendental equations
- 7. Numerical differentiation and integration
- 8. The numerical solution of ordinary differential equations
- 9. Curve fitting

SMITH, G. D.: Numerical Solution of Partial Differential Equations.

Oxford (1965) 179 pp.

This carefully written text is probably the most elementary book which is devoted exclusively to the numerical solution of partial differential equations. There are many exercises and worked examples.

- 1. Introduction and finite difference formulas
- 2. Parabolic equations
- 3. Convergence, stability, and systematic iterative methods
- 4. Hyperbolic equations and characteristics
- 5. Elliptic equations

SOUTHWELL, R. V.: Relaxation Methods in Engineering Science. Oxford,
Clarendon Press.

SOUTHWELL, R. V.: Relaxation Methods in Theoretical Physics (2 vols).

These are a fairly advanced series of books which were published from 1940 to 1956 as treatises on relaxation methods. The emphasis is always on the solution of practical engineering problems. There are especially many examples from the fields of hydrodynamics and elasticity. These methods have been used more by workers using desk computers than by high speed computer people, probably because of the complexity of the suitable digital computer programs. There are good bibliographies given.

1940 Volume

Relaxation methods applied to pin jointed and continuous girders

Plane and space frameworks having rigid joints; block relaxations

Theoretical aspects of the relaxation method

Electrical retworks; gyrostatic systems

Natural modes and frequencies of vibrating systems

Forced oscillations and the theory of dissipative forces

Continuous systems; problems of equilibrium; elastic stability

Nonlinear systems

Volume 1

Plane-potential and quasi-plane potential problems

The problem of conformal transformation

Problems involving boundaries or interfaces not initially known

Volume 2

Biharmonic problems

Two diagram technique

Eigenvalue problems; "normal" vibrations of mechanical and electrodynamic systems

Elastic stability of flat plate subjected to tractions

Nonlinear problems in elasticity, hydrodynamics, and plasticity

Three dimensional relaxation

Heat conduction and other transient problems

SOUTHWORTH, R. W. and DE LEEUW, S. L.: Digital Computation and

Numerical Methods. McGraw-Hill (1965) 508 pp.

This has been written as a textbook for a college freshman engineering course. It can be recommended over the rest of the present flood of books on computer programming and numerical analysis only because of its greater volume of material and because it gives some engineering applications. Ordinarily one would not expect to find so many somewhat more advanced topics in a freshman text. The programming language described is FORTRAN IV; the numerical analysis is classical.

- 1. Introduction to and history of computers
- 2. Flow charting
- 3. The FORTRAN language
- 4. Number systems and machine language
- 5. Rounding and truncation errors
- 6. Roots of equations
- 7. Simultaneous linear equations
- 8. Interpolation
- 9. Numerical differentiation and integration
- 10. Taylor's series
- 11. Numerical solution of ordinary differential equations
- 12. Empirical formulas and approximation

STANTON, R. G.: Numerical Methods for Science and Engineering. Prentice-

Hall (1961) 266 pp.

This contains a standard elementary treatment of numerical analysis for the college Junior; there are no particular applications to science or engineering, as the title might imply. Most of the topics are covered reasonably fully for the level to which the book is aimed, but (for example) the treatment of numerical solution of ordinary differential equations should have been modernized.

- 1. Ordinary finite differences
- 2. Divided differences
- 3. Central differences
- 4. Inverse interpolation and the solution of equations
- 5. Computation with series and integrals
- 6. Numerical solution of differential equations
- 7. Linear systems and matrices
- 8. Solution of linear equations
- 9. Difference equations
- 10. Solution of differential equations by difference equation methods
- 11. The principles of automatic computation

STEFFENSEN, J. F.: <u>Interpolation</u>, 2nd ed. Chelsea (1950) 248 pp.

This is a reasonably elementary text on interpolation formulas and their use in numerical differentiation, integration, summation, etc. It is a translation from the Danish edition, which came out in 1925. The subjects covered are each covered in quite some detail.

- 1. Displacement symbols and differences
- 2. Divided differences
- 3. Interpolation formulas
- 4. Some applications
- 5. Factorial coefficients
- 6. Numerical differentiation
- 7. Construction of tables
- 8. Inverse interpolation
- 9. Elementary methods of summation
- 10. Repeated summation
- 11. Laplace's and Gauss' summation formulas
- 12. Bernoulli polynomials
- 13. Euler's summation formula
- 14. Lubbock's and Woolhouse's formulas
- 15. Mechanical quadrature
- 16. Numerical integration of differential equations
- 17. The calculus of symbols
- 18. Interpolation with several variables
- 19. Mechanical cubature
- 20. On differential coefficients of arbitrary order

**STIEFEL, E. L.: An Introduction to Numerical Mathematics. Academic Press (1963) 286 pp.

This reasonably elementary textbook should be very popular among computer programmers who wish to better understand some of the numerical procedures which are used to solve scientific problems on computers. The choice of topics is particularly good for a person who is interested in the applications of numerical analysis. It is a translation from the German, but it does not seem to suffer from the translation difficulties so often noticeable when the translators are not completely familiar with the subject matter.

General Outline of Contents

- 1. Linear algebra
- 2. Linear programming
- 3. Least-squares approximation and definite problems
- 4. Nonlinear algebra
- 5. Eigenvalue problems
- 6. Differential equations (numerical differentiation and integration; first order differential equations and systems; boundary value problems; partial differential equations)
- 7. Approximations

SZEGÖ, G.: Orthogonal Polynomials, revised ed., Amer. Math. Soc., Colloquium Publications, No. 23 (1959) 421 pp.

This is an advanced treatise dealing with the general theory of orthogonal polynomials as well as with the study of certain classes of these polynomials. Numerical aspects are seldom stressed. There is an excellent list of references given.

- 1. Preliminaries
- 2. Definition of orthogonal polynomials; principal examples
- 3. General properties
- 4. Jacobi polynomials
- 5. Laguerre and Hermite polynomials
- 6. Zeros of orthogonal polynomials
- 7. Inequalities
- 8. Asymptotic properties of the classical polynomials
- 9. Expansion problems associated with the classical polynomials
- 10. Representation of positive functions
- 11. Polynomials orthogonal on the unit circle
- 12. Asymptotic properties of general orthogonal polynomials
- 13. Expansion problems associated with general orthogonal polynomials
- 14. Interpolation
- 15. Mechanical quadrature
- 16. Polynomials orthogonal on an arbitrary curve

THOM, A. and APELT, C. J.: Field Computations in Engineering and Physics.

Van Nostrand (1961) 168 pp.

This book discusses in a reasonably elementary manner the method of Squares, which has been used by Thom and others for nearly 40 years in the numerical solution of partial differential equation boundary value problems. The authors maintain that their methods of speeding convergence of the iteration processes are easier to program on a digital computer than are relaxation methods.

- 1. Finite difference formulas
- 2. Numerical solution of Laplace's equation
- 3. Symmetromorphic figures and free streamline problems
- 4. Numerical solution of Poisson's equation
- 5. Accuracy of solutions
- 6. Numerical solution of the equation for the flow of a compressible inviscid fluid
- 7. The biharmonic equations
- 8. Numerical solution of Navier-Stokes equations
- 9. Solution of problems in three dimensions with axial symmetry

TIMAN, A. F.: Theory of Approximation of Functions of a Real Variable.

Macmillan (1963) 631 pp.

This advanced treatise is a translation from a Russian volume published in 1960. In general, the book is similar to those by Achieser (originally published in 1947) and by Natanson (originally published in 1949), but it includes newer Russian research which had not been available to the earlier authors.

- 1. Weierstrass' theorem
- 2. The best approximation
- 3. Some compact classes of functions and their structural characteristics
- 4. Some properties of algebraic polynomials and transcendental integral functions of exponential type
- 5. Direct theorems of the constructive theory of functions
- 6. Converse theorems. Constructive characteristics of some classes of functions
- 7. Further theorems connecting the best approximations of functions with their structural properties
- 8. Linear processes of approximation of functions by polynomials and some estimates connected with them
- 9. Some results from the theory of functions and functional analysis

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TOCHER, K. D.: The Art of Simulation. Van Nostrand (1963) 184 pp.

This book is suggested for a computer user who wishes to learn more about the theory of the simulation and Monte Carlo problems which are to be carried out on the computer. The author assumes a basic knowledge of probability and statistics, but assuming this, the book could probably be classified as reasonably elementary.

Contents

1. Introduction

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- 2. Sampling from a distribution
- 3. More sampling methods
- 4. Random number tables
- 5. Random number generators
- 6. Pseudo-random numbers
- 7. Elementary sampling experiments
- 8. Flow diagrams
- 9. Estimation
- 10. Simple queuing problems
- 11. Further queue problems
- 12. General simulation problems
- 13. Design of simulation experiments

TODD, J.: Introduction to the Constructive Theory of Functions.

Academic Press (1963) 127 pp.

This is a somewhat advanced book, and is probably used in first year graduate courses. The title has reference to the approximate representation of functions in terms of simpler ones, but several other background topics take up some of the space, so that the term "introduction" in the title is really warranted. Also, pages 93 to 127 are taken up with solutions to problems, bibliography, etc. If one were studying this topic for the first time, he might wish to read this book and Natanson's books concurrently.

- 1. Results from algebra and analysis
- 2. The theorems of Weierstrass
- 3. The Chebyshev theory
- 4. The theorems of the Markoffs
- 5. Orthogonal polynomials
- 6. Interpolation and interpolation processes
- 7. Bernoulli polynomials
- 8. Function spaces
- 9. Approximate quadrature

**TODD, J. (Ed.): Survey of Numerical Analysis. McGraw-Hill (1962) 589 pp.

This book is based upon a series of lectures by the various authors in a training program in numerical analysis for senior university staff members who had not previously specialized in numerical analysis. Some of the chapters are reasonably elementary, while others are quite advanced. Most chapters contain excellent bibliographies, and this book is used extensively as a general reference work on modern numerical analysis. The index is skimpy, but the table of contents has a lot of detail; both should be consulted before giving up on finding a particular subject.

Contents

- 1. Motivation for working in numerical analysis (Todd)
- 2. Classical numerical analysis (interpolation; quadrature and differentiation; differential equations; etc.) (Todd)
- 3. The constructive theory of functions (Todd)
- 4. Autoratic computers (Newman and Todd)
- 5. Use and limitation of computers (Cohn)
- 6. Matrix computations (Newman)

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- 7. Numerical methods for finding solutions of nonlinear equations (Hochstrasser)
- 8. Eigenvalues of finite matrices (Taussky and Marcus)
- 9. Numerical methods in ordinary differential equations (Antosiewicz, Gautschi)
- 10. Orthonormalizing codes in numerical analysis (Davis)
- 11. The numerical solution of elliptic and parabolic partial differential equations (Young)
- 12. Numerical methods for integral equations (Bückner)
- 13. Errors of numerical approximation for analytic functions (Davis)
- 14. Numerical analysis and functional analysis (Antosiewicz and Rheinboldt)
- 15. Discrete problems (block design; group theory problems) (Hall)
- 16. Number theory (Cohn and Taussky)
- 17. Linear estimation and related topics (Zelan)

TRAUB, J. F.: <u>Iterative Methods for the Solution of Equations</u>. Prentice-Hall (1964) 310 pp.

This quite advanced book considers the problem of finding real roots of algebraic or transcendental equations or systems of equations. The author states that the extension of many of his results to the case of complex roots would not be difficult. Although parts of the book seem quite theoretical, the general tone is one of considering the efficiency of various iterative root finding schemes which might be used on a digital computer.

General Outline of Contents

- 1. Introduction and basic concepts and notations
- 2. General theorems on iteration functions (convergence theorems and the iteration calculus)
- 3. The mathematics of difference relations (intermediate theorems on solutions of certain difference equations and on the asymptotic behavior of some solutions)
- 4. Iteration functions generated by direct or inverse hyperosculatory interpolation
- 5. One-point iteration functions
- 6. One-point iteration functions with memory (using old information concerning previous iterates)
- 7. Concerning multiple roots
- 8. & 9.: Multi-point iteration functions (there are no well known examples of such iteration functions)
- 10. Iteration functions which require no evaluation of derivatives
- 11. Systems of equations
- 12. A compilation of iteration functions

Appendices

Interpolation

On the jth derivative of the inverse function
Significant figures and computational efficiency
Acceleration of convergence
Numerical examples
Areas for future research

- VAJDA, S.: The Theory of Games and Linear Programming. Wiley (1957) 106 pp.
- VAJDA, S.: Readings in Linear Programming, 2rl ed. Wiley (1962) 130 pp.

These two books give quite an elementary introduction into the field of linear programming. The first named book has many worked examples, and the second book is practically a listing of various types of "practical" problems. Neither book requires much mathematics of the reader.

Contents of the First Book

- 1. An outline of the theory of games
- 2. Graphical representation
- 3. Algebra and theory of games
- 4. An outline of linear programming
- 5. Graphical representation of linear programming
- 6. Algebra of the simplex method
- 7. Degeneracy and other complications
- 8. Duality
- 9. The solution of games
- 10. The method of leading variables

Contents of the Second Book

Transportation problems, caterer problem, production scheduling, transshipment, bid evaluation, flow through a network, ship scheduling, personnel assignment (allocation), routing aircraft, investment; the simplex tableau; nutrition problem, airlift, blending of gasolines, smooth patterns of production; duality; selection of products, trim loss reduction, attendants' rota, warehousing, games. **VARGA, R.: Matrix Iterative Analysis. Prentice-Hall (1962) 322 pp.

This somewhat advanced book discusses the numerical solution of matrix problems arising from discrete approximations to elliptic partial differential equations. Although the mathematical theory is there, the whole point of the book is to show how these particular large linear systems can be solved efficiently on large scale digital computers. There is also a chapter on matrix methods for problems arising from discrete approximations to parabolic partial differential equations. There is a 13-page bibliography.

- Matrix properties and concepts
- 2. Non-negative matrices
- 3. Basic iterative methods and comparison theorems
- 4. Successive overrelaxation iterative methods
- 5. Semi-iterative methods
- 6. Derivation and solution of elliptic difference equations
- 7. Alternating direction implicit iterative methods
- 8. Matrix methods for parabolic partial differential equations
- 9. Estimation of acceleration parameters

VILENKIN, N. Ya.: Successive Approximation. Macmillan (1964) 70 pp.

This very elementary book is a translation from a 1961 Russian edition. Most methods are explained graphical means, so the book might be of some interest, even for those who could still understand a somewhat more difficult book. There is no index.

- 1. Introduction
- 2. Successive approximations
- 3. Achilles and the tortoise
- 4. Division on digital computers
- 5. Extracting square roots by successive approximation
- 6. Extracting kth roots by successive approximation
- 7. The derivative of a polynomial
- 8. Newton's method for the approximate solution of algebraic equations
- 9. The geometric interpolation of the derivative
- 10. The geometric interpretation of Newton's method
- 11. The derivative of more general functions
- 12. The calculation of derivatives
- 13. Finding a first approximation
- 14. The method of chords
- 15. The method of iteration
- 16. The geometric interpretation of the method of iteration
- 17. The speed of convergence of a sequence of successive approximations
- 18. The solution of a system of linear equations by successive approximation
- 19. Successive approximations in geometry
- 20. Conclusion

WALSH, J. L.: <u>Interpolation and Approximation by Rational Functions in</u>

the Complex Domain, 2nd ed. Amer. Math. Soc., Colloquium Publications
No. 20 (1956) 398 pp.

This is an advanced treatise. The theoretical aspects of the field are well covered here.

- 1. & 2.: Possiblity of approximation; analytic functions
- 3. Interpolation and lemniscates
- 4. Degree of convergence of polynomials; overconvergence
- 5. Best approximation by polynomials
- 6. Orthogonality and least squares
- 7. Interpolation by polynomials
- 8. Interpolation by rational functions
- 9. Approximation by rational functions
- 10. Interpolation and functions analytic in the unit circle
- 11. Approximation with auxiliary conditions and to nonanalytic functions
- 12. Existence and uniqueness of rational functions of best approximations

WHITTAKER, E. T. and ROBINSON, G.: The Calculus of Observations, 3rd ed.

Blackie and Son (1940) 395 pp.

This treatise is a little more advanced in parts than some of the more modern texts, but generally it is quite readable. It is often quoted as a reference. There are several chapters discussing the numerical aspects of certain statistical subjects.

Contents

- 1. Interpolation with equal intervals of the argument
- 2. Interpolation with unequal intervals of the argument
- 3. Central differences formulas
- 4. Applications of difference formulas
- 5. Determinants and linear equations
- 6. Numerical solution of algebraic and transcendental equations
- 7. Numerical integration and summation
- 8. Normal frequency distributions
- 9. The method of least squares
- 10. Practical Fourier analysis
- 11. Graduation, or the smoothing of data
- 12. Correlation
- 13. The search for periodicities
- 14. The numerical solution of differential equations
- 15. Further problems

Summation of slowly convergent series

Prony's method of interpolation by exponentials

Interpolation formulas for functions of two arguments

Numerical computation of double integrals

Numerical solution of integral equations

Rayleigh-Ritz method for minimum problems

Application to the determination of eigenvalues

**WILKINSON, J. H.: The Algebraic Eigenvalue Problem. Oxford (1965) 662 pp.

This new book has been eagerly awaited by those who are trying to solve such problems on digital computers. The subject is rather advanced, but explanations are in general unusually clear, and the book should be of great value to knowledgeable engineers and computer programmers as well as to specialists in the field. The author gives comparative assessments of various techniques for solving the algebraic eigenvalue problem.

- 1. Theoretical background
- 2. Perturbation theory
- 3. Error analysis
- 4. Solution of linear algebraic equations
- Hermetian matrices
- 6. Reduction of a general matrix to condensed form
- 7. Eigenvalues of matrices of condensed forms
- 8. The LR and QR algorithms
- 9. Iterative methods

**WILKINSON, J. H.: Rounding Errors in Algebraic Processes. Prentice-Hall (1963) 161 pp.

This definitive text on the subject is of intermediate difficulty. It is written in such a clear manner that a computer user with some slight knowledge of numerical analysis could improve his intuitive knowledge on the effect of round-off in various computing processes without actually working through all the details of the various proofs. Upper bounds are given in many cases; the more difficult problem of obtaining a (more realistic) statistical bound is usually not covered.

Contents

- 1. Rounding errors in arithmetic operations (for both fixed point and floating point cases); ill-conditioned problems
- Computations involving polynomials
 Evaluation of polynomials; finding zeros of polynomials; ill-conditioned polynomials
- 3. Matrix computations

Error analysis of simple matrix operations; orthogonalization of vectors; solution of systems of linear equations; matrix inversion. Error analysis of Gaussian elimination; both floating point and fixed point bounds. Determinant evaluation; solutions of triangular equations; inversion of matrices; left-handed and right-handed inverses; use of an estimate for $||A^{-1}||$; calculation of Eigenvalues and Eigenvectors; ill-conditioned Eigenvalues

WILLERS, F. A.: Practical Analysis. Dover (1955) 422 pp.

This is a reasonably elementary treatise on graphical and numerical methods. It is a translation from the German of a 1928 text, but it does, however, have new sections describing American desk calculators, slide rules, etc. There is no discussion of high speed digital computers.

- Numerical calculation and its aids (graphs, slide rules, nomograms, calculating machines)
- Interpolation (interpolation formulas; numerical differentiation and integration; cubature)
- 3. Approximate integration and differentiation (graphical methods; the planimeter; numerical methods)
- 4. Practical equation theory (roots of equations and systems of equations)
- 5. Analysis of empirical function (approximations by linear functions, rational integral functions, periodic functions, and exponential functions)
- 6. Approximate integration of ordinary differential equations (graphical methods, numerical methods, and mechanical integrators)

ZAGUSKIN, V. L.: <u>Handbook of Numerical Methods for the Solution of</u>

Algebraic and Transcendendental Equations. Pergamon (1961) 195 pp.

This translation from the (1960) Russian edition presents an elementary exposition of these topics. Digital computer applications are considered in many of the choices of subject matter which have been made.

General Outline of Contents

- 1. Introductory material about polynomials and transcendental functions: Zeros of a function; basic properties of polynomials; changing the argument of a polynomial; polynomials with complex coefficients; algorithms for multiplication and division of polynomials; Horner's method; position of zeros of a polynomial
- 2. Operations with approximate numbers Absolute and relative errors; round off; error in calculating value of a polynomial; solution of approximate equation; etc.
- 3. Methods for approximate determination of roots Graphical method; Viet's formulas; Lobachevskii method; Bernoulli method; method of iteration; Lin method; Paluver method; comparison of methods
- 4. Methods of increasing the accuracy of roots already found Method of linear interpolation; Newton's method; Berstoi's method; Belostotskii's method; iteration with quadratic convergence; methods of improving convergence; comparison of methods
- 5. Solution of equations of low orders and extraction of roots

 Quadratic, cubic, quartic, and quintic equations; Newton's method;

 iteration with quadratic convergence
- 6. Solution of simultaneous equations

 Method of elimination; graphical method; method of iteration; Newton's method; method of steepest descent; comparison of methods of solution of simultaneous nonlinear equations; methods of solution of simultaneous linear equations

ZOUTENDIJK, G.: Methods of Feasible Directions. Elsevier (1960) 127 pp.

This fairly advanced book explains the author's particular method, which is sometimes used (perhaps in modified form) to solve nonlinear programming problems. There is also some preliminary discussion of other methods for linear or convex programming, but the main use of the book is in explaining the author's own methods.

- 1. Introduction
- 2. Theory of convex programming
- 3. Methods of solution for the linear programming problem
- 4. Computational algorithms for the simplex and dual simplex method
- 5. Numerical comparison of the different algorithms
- 6. Some special linear programming problems
- 7. Methods of feasible directions
- 8. Normalization of the feasible directions
- 9. The linear programming problem and the methods of feasible directions
- 10. Quadratic programming
- 11. Convex programming

Tracts for Computers, I.- . Cambridge Univ. Press (1919-).

These are a series of pamphlets published as "first aid" to the professional computer. They mostly consist of tables, but the five marked below with the symbol † are expository articles on the given subjects.

Titles

- I. Tables of Digamma and Trigamma functions (Pairman, 1919)
- † II-III. On the construction of tables and on interpolation (Pearson, 1920)
 - IV. Tables of logarithms of the complete Γ function to 12 figures. (Legendre, 1921)
 - V. Table of coefficients of Everett's central difference formula (Thompson, 1921)
- + VI. Smoothing (Rhodes, 1921)
- † VII. Numerical evaluation of the incomplete β function or of the integral $\int_0^{\mathbf{x}} \mathbf{x}^{\rho-1} (1-\mathbf{x})^{q-1} d\mathbf{x}$ for ranges of \mathbf{x} between 0 and 1 (Soper, 1921)
 - VIII. Table of logarithms of the complete [function (for arguments 2-1200, i.e., beyond Legendre's range). (Pearson, 1922)
 - IX. Log $\Gamma(x)$ from x = 1 to 50.9 by intervals of 0.01 (Brownlee, 1923)
- †X. On quadrature and cubature (Irwin, 1923)
- XI. Logarithmetica Britannica, being a standard table of logarithms to 20 decimal places. Part IX, No's. 90,000 to 100,000 (Thompson)
- XII. Tables of probable error of the coefficient of correlation as found by the product moment method (Holzinger, 1925).
- XIII. Biblioteca Tabularum Mathematicarum, being a descriptive catalog of mathematical tables. Part 1, Logarithmic tables (Henderson (1926)
- XIV. & XVI. XXII.: Logarithmetica Britannica, Parts I to VIII, No's. 10,000 to 90,000 (Thompson)
- XXIII. Tables of $tan^{-1}x$ and $log(1+x^2)$. (Comrie. 1938)
- XXIV. Tables of random sampling numbers (Kendall and Smith, 1939)
- XXV. Random normal deviates (Wold, 1948)
- XXVI. Correlated random normal deviates (Fieller, Lewis, and Pearson (1955)
- XXVII. Random negative exponential deviates (Barnett, 1965)

NATIONAL PHYSICAL LABORATORY: <u>Mathematical Tables Series 1 - .</u> Her Majesty's Stationery Office. London (1956-)

As the title indicates, these pamphlets are mostly tables. However, the two marked below with the symbol " are mainly expository articles on the subjects. Also, even if the major portion of a volume is a table, there is some explanatory material given. The titles in the set are:

- *1. The use and construction of mathematical tables (Fox, 1956)
- 2. Tables of Everett interpolation coefficients (Fox, 1958)
- 3. Tables of generalized exponential integrals (Miller, 1960)
- 4. Tables of Weber parabolic cylinder functions and other functions of large arguments (Fox, 1960)
- †5. Chebyshev series for mathematical functions (Clenshaw, 1962)
- 6. Tables of Bessel functions of moderate or large orders (Olver, 1963)
- 7. Tables of Jacobian elliptic functions whose arguments are rational functions of the quarter period (Curtis, 1964)

**NATIONAL BUREAU OF STANDARDS (U. S. Dept. Commerce): Applied Mathematics

Series 1-58 (1948-1964).

These publications are mostly tables. However, No's. 12, 15, 18, 24, 29, 33, 38, 39, 42, 47, 49, 54, and 57 contain only expository articles, and several others (also marked with the symbol †) have considerable explanatory material. This series was set up as a continuation of the previous table series which went from MT1 to MT37. This series and its predecessor are of the highest popularity among computing people.

Titles

- 1. Tables of the Bessel Functions $Y_0(x), Y_1(x), K_0(x), K_1(x), 0 \le x \le 1$. 60 pp.
- 2. Table of coefficients for obtaining the first derivative without differences (Salzer) 20 pp.
- 3. Table of the confluent hypergeometric function F(n/2,1/2;x) and related functions. 73 pp.
- 4. Tables of scattering functions for spherical particles. 119 pp.
- Tables of sines and cosines to 15 decimal places at hundredths of a degree. 95 pp.
- 6. Tables of the binomial probability distribution. 387 pp.
- †7. Tables to facilitate sequential t-tests. 82 pp.
- 8. Table of powers of complex numbers (Salzer) 44 pp.
- ²9. Tables of Chebyshev polynomials $S_n(x)$ and $C_n(x)$. 161 pp.
- Tables for conversion of x-ray diffraction angles to interplanar spacing.
 pp.
- 11. Tables of arctangents of rational functions (Todd) 105 pp.
- †12. Monte Carlo method (Proceedings of a symposium conducted in 1949). 42 pp.
- 13. Tables for the analysis of beta spectra. 61 pp.
- 14. Tables of the exponential function e^{x} . 537 pp.
- +15. Problems for the numerical analysis of the future (four papers from a 1948 symposium). 21 pp.
- 16. Tables of n! and $\Gamma(n+1/2)$ for the first thousand values of n (Salzer) 10 pp.
- †17. Tables of Coulomb wave functions I. 141 pp.

- †18. Construction and application of conformal maps (Proceedings of a symposium conducted in 1949) 280 pp.
- †19. Hypergeometric and Legendre functions with applications to integral equations of potential theory (Snow) 427 pp.
- †20. Tables for rocket and comet orbits (Herrick) 128 pp.
- †21. A guide to tables of the normal probability integral. 16 pp.
- †22. Probability tables for the analysis of extreme-value data. 32 pp.
- 23. Tables of normal probability functions $\frac{1}{\sqrt{2\pi}} e^{-x/2}$ and $\frac{1}{\sqrt{2\pi}} \int_{-x}^{x} e^{-\alpha^2/2} d\alpha$
- †24. Introduction to the theory of stochastic processes depending on a continuous parameter (Mann) 45 pp.
- 25. Tables of the Bessel functions $Y_0(x), Y_1(x), K_0(x), K_1(x), 0 \le x \le 1$. AAS1) 60 pp.
- 26. Table of arctan x. 17 pp.
- 27. Tables of 10^x. 543 p.
- 28. Tables of Bessel-Clifford functions of orders zero and one. 72 pp.
- **†29. Simultaneous linear euqations and the determination of eigenvalues (Paige and Taussky) 126 pp.
 - 30. Tables of coefficients for the numerical calculation of Laplace transforms (Salzer) 36 pp.
 - 31. Table of natural logarithms for arguments between zero and five to sixteen decimal places. 501 pp.
 - 32. Table of sine and cosine integrals for arguments from 10 to 100. 187 pp.
 - †33. The statistical theory of extreme values and some practical applications (Gumbel) 51 pp.
 - 34. Table of the gamma function for complex arguments. 105 pp.
 - 35. Tables of Lagrangian coefficients for sexagesimal interpolation.
 157 pp.
 - 36. Tables of circular and hyperbolic sines and cosines. 407 pp.
 - †37. Tables of functions and zeros of functions. 211 pp.
 - †38. Magnetic fields of cylindrical coils and annular coils (Snow) 29 pp.

- **+39. Contributions to the solution of systems of linear equations and the determination of eigenvalues (Taussky) 139 pp.
 - 40. Tables of secants and cosecants to nine significant figures at hundredths of a degree. 46 pp.
 - 41. Tables of the error function and its derivative. 302 pp.
 - †42. Experiments in the computation of conformal maps (Todd) 61 pp.
 - 43a. Tables of sines and cosines for radian arguments. 278 pp.
 - 44. Table of salvo kill probabilities for square targets. 33 pp.
 - 45. Table of hyperbolic sines and cosines x = 2 to x = 10. 81 pp.
 - 46. Table of the descending exponential x = 2.5 to x = 10. 76 pp.
 - †47. Contributions on partially balanced incomplete block designs with two associate classes (Clatworthy) 76 pp.
 - †48. Fractional factorial experiment designs for factors at two levels. 85 pp.
- **+49. Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues.
 - 50. Tables of the bivariate normal distribution function and related functions. 258 pp.
 - 51. Tables of the exponential integral for complex arguments. 634 pp.
 - 52. Integrals of airy functions
 - 53. Table of natural logarithms for arguments between five and ten to sixteen decimal places. 506 pp.
 - †54. Fractional factorial experiment designs for factors at three levels (Conner and Zelen) 37 pp.
- **+55. Handbook of Mathematical Functions (Abramowitz and Stegun). 1046 pp.
 - 56. Tables of osculatory interpolation coefficients (Salzer) 25 pp.
- **+57. Basic theorems in matrix theory (Marcus) 27 pp.
 - 58. Fractional factorial designs for experiments with factors at two and three levels (Conner and Young, 65 pp.)

⁽A double asterisk ** by an author's name means that the reference is considered especially good and/or popular for the average scientific computer user who is required to learn about the subject.)

PART II. HOW TO FIND WHAT YOU WANT

These paragraphs are written for those who have had very little experience in looking up scientific information in the literature, as well as for the more experienced scientists who are newcomers to the ranks of computer users. How does one find something in the literature of this field? How does he find whether there is any literature on a given numerical subject? How does he find whether or not certain desired tables exist?

A. BIBLIOGRAPHIES, LISTS OF BOOKS, ABSTRACTING JOURNALS

1. <u>General</u>: If one knows absolutely nothing about a subject, he probably should first see if he can find it discussed in a textbook. Perhaps Part I of this document would help. For a less specialized but older listing, one might consider

PARKE III, N. G.: Guide to the Literature of Mathematics and

Physics, Including Related Works in Engi
neering Science, 2nd ed. Dover (1958) 436 pp.

This includes references to the periodical literature (in all languages) as well as to books. There is a short discussion of the literature in many of the subfields into which the basic subjects have been divided. A still older list, which has been made up especially for a computer laboratory, is

FORSYTHE, G. E.: "A Numerical Analyst's Fifteen-Foot Shelf,"

M.T.A.C. 7 (1953) pp. 221-228.

This list includes 150 essential titles under the headings: (1) bibliographies, (2) collections of formulas, (3) references on numerical methods, (4) references on applied mathematics, (5) references on pure mathematics. It includes outstanding foreign language references.

2. <u>Abstracting Journals:</u> The best way to be reasonably sure not to miss out on any possible references is to look through the abstracting journals in the field. The best known abstracting journals used by computer people are listed on the following page.

- **Mathematical Reviews, Vol. 1- , Amer. Math. Soc., New York (1940-)
 - Computer Abstracts, Vol. 1 , Technical Information Co., Ltd., Chancery House, London (1957)
- **Computing Reviews, Vol. 1 , Association for Computing Machinery, (1960-).

New books and periodical articles are reviewed in each of these periodicals. A complete reference as to where to find the article under review is given, and the reviewer tells a little bit about what is in the article or book. Mathematical Reviews covers all branches of mathematics and related fields. Computer Abstracts and Computing Reviews discuss references on computers and related fields. Computing Reviews publishes an extremely useful cumulative KWIC index periodically in which all reviewed articles are listed by review number, author, and by subject (key word in title). Some books or articles are reviewed in all three abstracting journals, while others may be reviewed in only one or two of them; it depends on the subject matter.

These abstracting journals are quite complete. For example, if in 1966 a person carefully looks through the yearly indexes of Mathematical Reviews for a certain mathematical topic, he has hopes that he has obtained most of the pertinent references to the subject which were published anywhere in the world in the 25-year period, 1939-1964. To make a thorough search of the earlier mathematical literature, one would have to go to the German abstracting journals

Jahrbuch über die Fortschritte der Mathematik, Vol. 1 - 60. de Gruyter, Berlin (1868-1934)

Zentralblatt fur Mathematik, Vol. 1 - , Springer, Berlin (1931-). Somewhat off the general subject of this document is the abstracting journal

Quality Control and Applied Statistics, Vol. 1 - . Interscience and Executive Sciences Inst. (1956-).

This monthly publication includes reviews of some papers discussing linear programming and related subjects. Another similar monthly abstracting journal is

Operations Research/Management Science, Vol. 1 - . Executive Sciences Inst. (1961-).

3. More Specialized Bibliographies and Annotated Bibliographies. Many of the books discussed in Part I have very complete bibliographies. There is a 15-page bibliography on numerical analysis (emphasizing matrices) in Householder's book (reviewed on p.). A supplementary list is given in the article

HOUSEHOLDER, A. S.: "Bibliography on Numerical Analysis," J. of A.C.M. 3,2 (1956) pp. 85-100.

A very specialized list is given in the reference,

STROUD, A. H.: "A Bibliography on Approximate Integration," Math of Comp. 15,73 (1961) pp. 52-80.

This bibliography lists papers dealing with quadrature formulas (using weighted linear sums of the values of the integrand and possibly derivatives of the integrand) and with finite difference approximations. It does not list papers on applications to the solution of differential equations or papers on graphical methods. Another specialized listing is given in the article

FORSYTHE, G. E.: "Tentative Classification of Methods and Bibliography on Solving Systems of Linear Equations," Appl. Math.

Series No. 29, Simultaneous Linear Equations and the Determination of Eigenvalues. National Bureau of Standards, Washington, D.C. (1953).

This 28-page article is an excellent place to start in the detailed study of this subject. Another specialized listing is given in the book

SHOHAT, J. A., HILLE, E., and WALSH, J. L.: A Bibliography on Orthogonal Polynomials. Bulletin of National Research Council, No. 103. Washington, D. C. (1940) 204 pp.

This very cleverly worked out bibliography uses a code symbol system so as to indicate content and, in many cases, results of individual papers. There is a short topical index.

The bibliography

YOUDEN, W. W.: <u>Computer Literature Bibliography 1946-1963</u>. National Bureau of Standards Misc. Publ. 266 (1965) 463 pp.

lists all papers in 9 journals, 100 proceedings, and 21 multi-author books. They are listed in three ways: according to source, author, and subject.

A comprehensive earlier annotated bibliography on linear, nonlinear, and dynamic programming is given in

RILEY, V. and GASS, S. I.: <u>Linear Programming and Associated</u>

<u>Techniques</u>. Johns Hopkins Press (1958) 613 pp.

A quite recent 20-page listing which covers only the subjects of Quadratic Programming, Nonlinear Programming, Programming under Uncertainty, and Integer Programming (and which gives no discussion of the papers) is

COTTLE, R. W., and VAN SLYKE, R. M.: <u>Bibliography on Mathematical</u>

<u>Programming</u>. Univ. Calif. Operations Research Center Report
64-6 (RN). (March 1964).

Other works in related fields are reviewed in

BATCHELOR, J. H.: Operations Research, an Annotated Bibliography. (1959) 1; (1962) 2; (1963) 3; (1964) 4. St. Louis Acad. Press (Vol. 4 includes most references published in 1961).

An earlier bibliography (with no discussion of the papers) is

CASE INSTITUTE OPERATIONS GROUP: Comprehensive Bibliography on Operations Research (through 1956 with supplement for 1957). Wiley (1958) 188 pp.

A complete bibliography on symbolic logic is found in

CHURCH, A.: "A Bibliography of Symbolic Logic," J. Sym. Logic 1 (1936) pp. 121-218. Additions and corrections, Ibid. 3 (1938) pp. 178-212.

Reviews of later articles have been published each year in the same journal.

4. <u>Indexes of Publications of Particular Groups</u>. There are a large number of such indexes. For example, there are several volumes (irregularly issued) of

Index of NACA Technical Publications. 1915 - date.

For documents acquired by the Defense Documentation Center, this organization publishes twice monthly a huge abstracting journal

Technical Abstract Bulletin (TAB).

Indexes and subject indexes to TAB are also available, so that one has some chance of finding what he wants. We will only list two other indexes of this general type:

RAND CORP.: Index of Selected Publications of the Rand Corporation, Vol. 1 (1946-62)

Selected Rand Abstracts (yearly), Vol. 1 - . 1963-

U.S. NATIONAL BUREAU OF STANDARDS: Publications 1901 - June 30, 1947.

NBS Circular 460, U.S. Govt. Printing Office (1948) 375 pp.

Supplementary List July 1, 1947 to June 30, 1957, 373 pp.

Publications of NBS July 1, 1957 to June 30, 1960, 391 pp.

Supplementary List July 1, 1960 to Feb. 26, 1965, 50 pp.

B. LOOKING FOR TABLES OF VARIOUS FUNCTIONS

There are two major sources where one can get information about a huge number of tables. FLETCHER, MILLER, and ROSENHEAD (reviewed on p. 33) is the "bible" in this regard. For the newer tables one looks in the periodical

Mathematics of Computation, Vol. 14 - . (Formerly Mathematical Tables and Other Aids to Computation (M.T.A.C.), Vol. 1 - 13)

National Research Council, Washington D. C. (1943-).

It of course takes quite a while to look through all the yearly volumes of this periodical, but it is the best place to find about tables published too recently to be listed in Fletcher, Miller, and Rosenhead.

Other good sources are LEBEDEV and FEDOROVA (reviewed on page 87) and SCHUTTE (reviewed on page 126).

For somewhat more specialized (statistical type) tables, one may consult

GREENWOOD, J. A. and HARTLEY, H. O.: <u>Guide to Tables in Mathematical Statistics</u>. Princeton Univ. Press (1962) 1014 pp.

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C. KEEPING UP WITH SOME OF THE NEW LITERATURE

Quite often the author has been questioned as to what periodicals he would recommend to a computer programmer who would like to get a little knowledge of what is being done right now in the field of numerical analysis and related topics. This of course depends a lot upon his background and interests. However, we will list some periodicals which we think are most popular among such people. If there are any of these the reader has never seen, it is recommended that he at least look at one copy to see what it is like. Then perhaps he will want to regularly read or glance through one or more of these.

**Mathematics of Computation (Quarterly)

**Numerische Mathematik (Quarterly, with about half the articles in English)

**SIAM Journal on Applied Mathematics (Quarterly)

**SIAM Journal on Numerical Analysis

SIAM Review (Quarterly)

Journal of the Association for Computing Machinery (Quarterly)

Communications of the Association for Computing Machinery (Monthly)

Computer Journal (Quarterly)

Nordisk Tidskrift for Informations-Behandling (BIT) (Quarterly; with most articles in English)

U.S.S.R. Computational Mathematics and Mathematical Physics (Bimonthly; cover-to-cover translations of a Russian periodical on the subject)

Journal of Mathematics and Physics (Quarterly)

Journal of Research, U. S. National Bureau of Standards (Series B, Mathematics and Mathematical Physics) (Quarterly)

I.B.M. Journal of Research (Quarterly)

** Mathematical Reviews (Monthly)

Computer Abstracts (Monthly)

**Computing Reviews (Bimonthly)

**Current Contents of Space and Physical Sciences (Weekly)

Those journals marked with a double asterisk are especially recommended. If the author were limited to seeing four periodicals regularly, he would choose Mathematics of Computation, Numerische

Mathematik, Siam Journal on Numerical Analysis, and Computing Reviews. Many computer people might want to read Mathematical Reviews or Computer Abstracts as abstracting bibliographies, instead of Computing Reviews.

The weekly publication, Current Contents of Space and Physical Sciences, 1961 - , contains the contents pages of over 500 foreign and domestic journals. If one took the time to go through this every week, he would have all the new articles brought to his attention quicker than in any other way we know. Most of the magazines are of course on other fields, so it would usually only take five or ten minutes to look through one week's edition for articles of interest to computer people. Of course, the articles are not reviewed here (as they are in Mathematical Reviews, Computer Abstracts, or Computing Reviews), and one has so judge from the title whether he wants the library to get the actual periodical for him.

PART III. INDEX

The numbers in this quite limited index refer to the pages of this document. Usually when more than five references are listed for a certain subject, some of the references are underlined. If one wishes a single general reference and is overwhelmed with the number of references given, the underlined references might be considered first: they generally contain quite a discussion of the given subject. For references consisting of more than one volume, this index usually includes the volume number. For example, 107v2 refers to vol. 2 of the reference listed on page 107 of this document. As another example, 156 (#39) means vol. 39 of the series described on page 156.

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